Discrete time formulations and valid inequalities for a maritime inventory routing problem

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Outline

- Description of Maritime Inventory Routing Problem(MIRP)
- A discrete time model
- A Fixed Charge Network Flow (FCNF) model
- Relaxation sets and valid inequalities
  - Knapsack and Mixed integer routing
  - Lot-sizing
- Computational study
- Concluding remarks

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Problem description

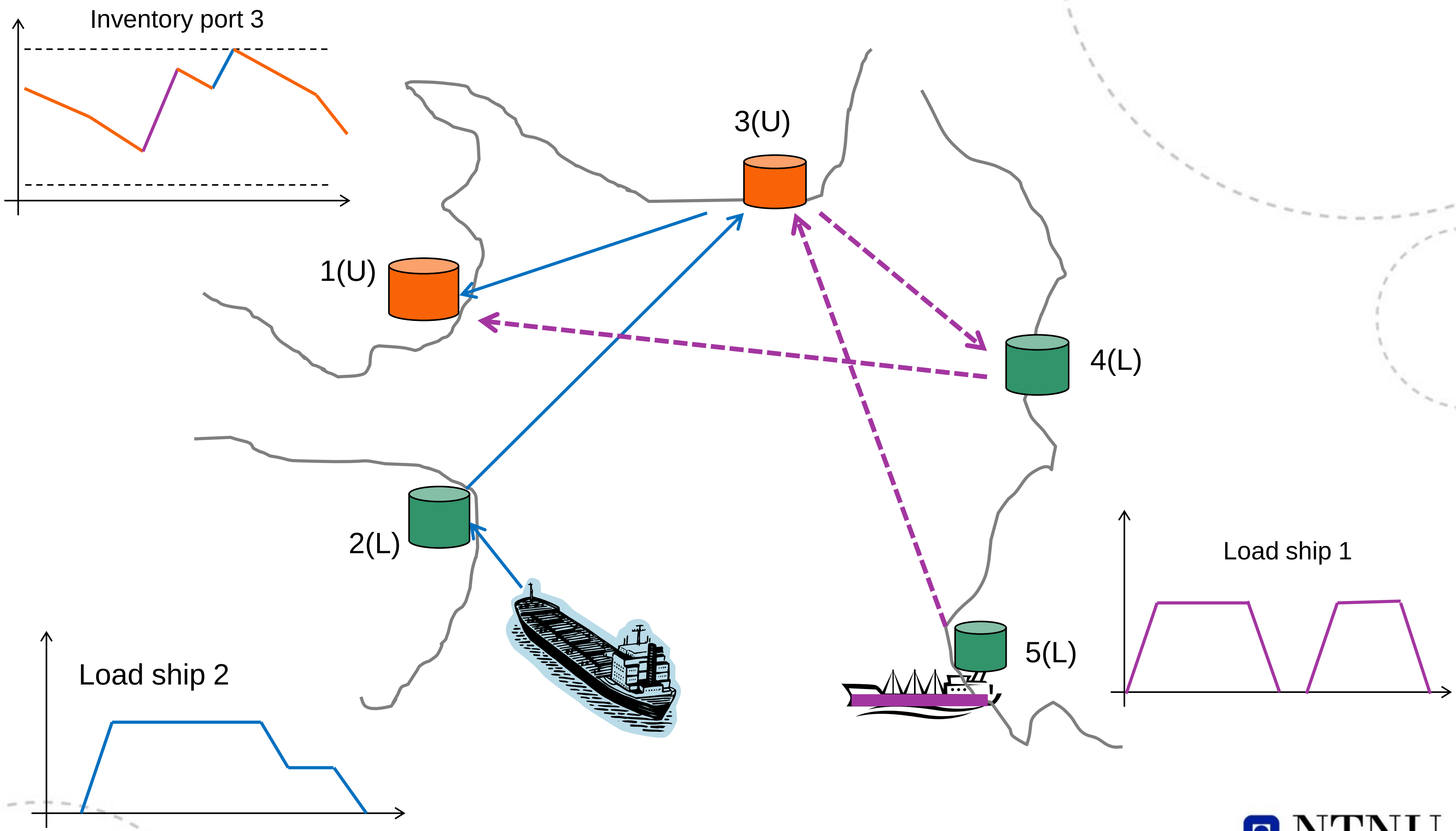
- Short sea shipping with long (un)loading times relative to sailing times
- A single bulk product

- Multiple production and consumption ports
- Time varying demand and production
- Limited storage capacity and safety stocks at the ports
- (Un)loading capacity per time period dependent on ship
- Berth capacities

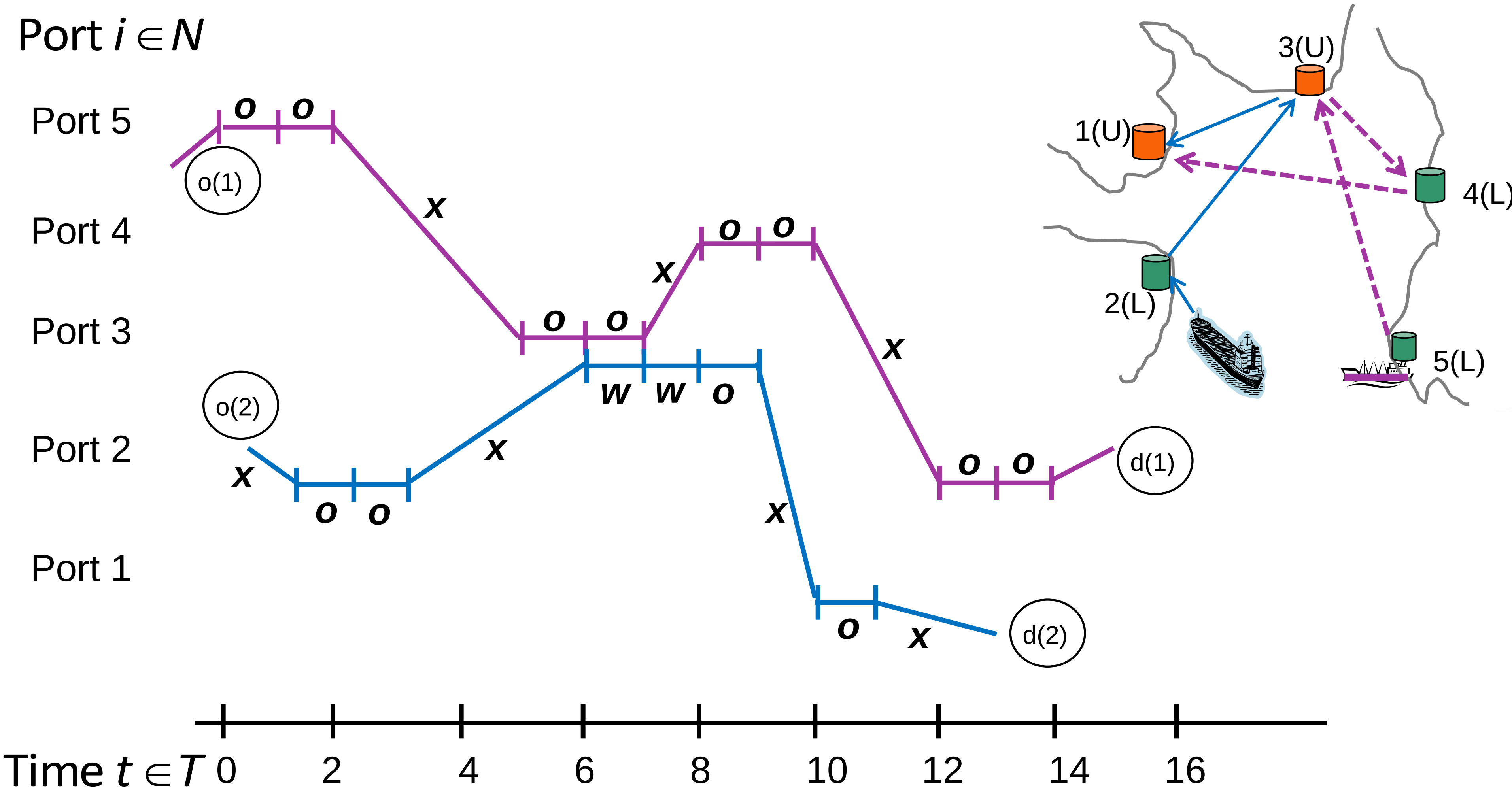
- A heterogeneous fleet of ships with given capacities
- Initial conditions for the ships are known
- Multiple (un)loadings in succession
- Waiting outside a port before (un)loading is possible

- Design routes and schedules minimizing the sailing and port costs and determine the loading quantity at each port without exceeding the storage capacities

Maritime inventory routing: Example



Ship paths in a time expanded network



Ship path in a time expaneded network

Port $i \in N$

Port 5

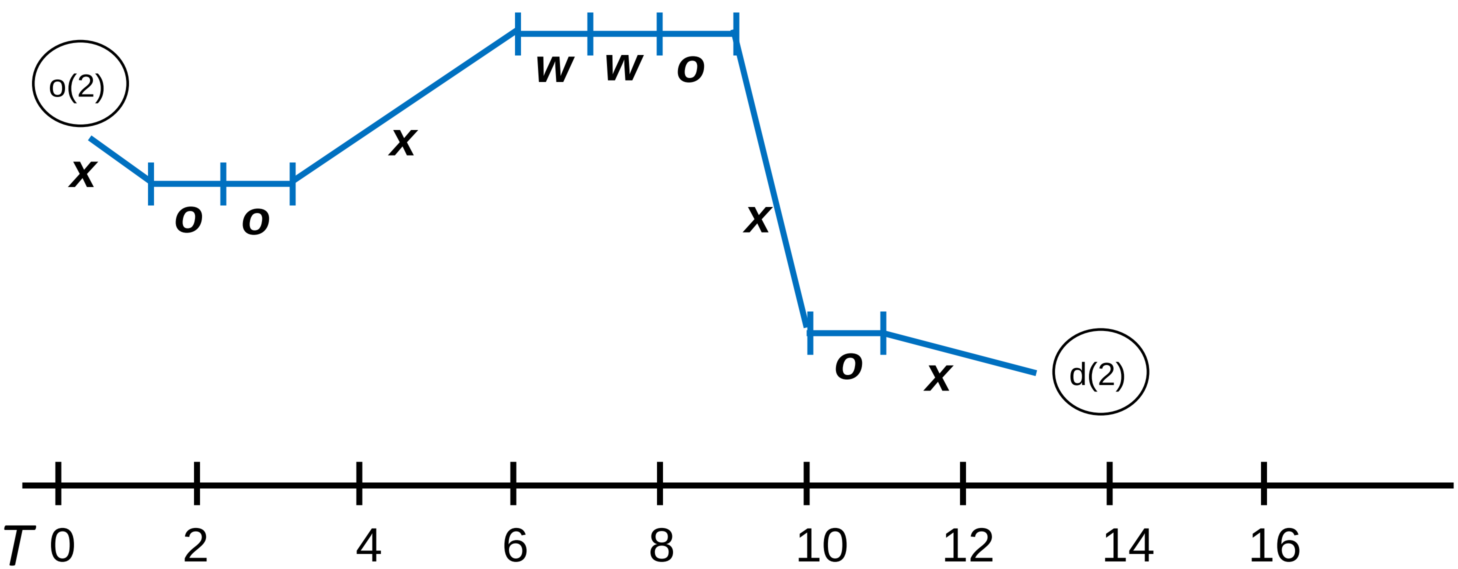
$$x_{ijvt}\,, \quad o_{ivt}\,, \quad w_{ivt}\,, \quad q_{ivt}\,, \quad l_{vt}\,, \quad s_{it}$$

Port 4

Port 3

Port 2

Port 1



Time $t \in T$

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Original formulation

$$\min \sum_{v\in V} \sum_{i\in N\cup\{o(v)\}} \sum_{j\in N\cup\{d(v)\}} \sum_{t\in T} C^T_{ijv} x_{ijvt} + \sum_{v\in V}\sum_{i\in N}\sum_{t\in T} C^P_{iv} o_{ivt} + \sum_{v\in V}\sum_{i\in N}\sum_{t\in T} C^W_v w_{ivt},$$

subject to:

$$\sum_{j\in N\cup\{d(v)\}} \sum_{t\in T} x_{o(v)jvt} = 1, \quad v\in V,$$

$$\sum_{i\in N\cup\{o(v)\}} \sum_{t\in T} x_{id(v)vt} = 1, \quad v\in V,$$

$$\sum_{j\in N\cup\{o(v)\}} x_{jiv,t-T_{jiv}} + w_{iv,t-1} + o_{iv,t-1} = \sum_{j\in N\cup\{d(v)\}} x_{ijvt} + w_{ivt} + o_{ivt}, \quad v\in V, i\in N, t\in T,$$

-Flow conservation

$$o_{iv,t-1} \le \sum_{j\in N\cup\{d(v)\}} x_{ijvt} + o_{ivt}, \quad v\in V, i\in N, t\in T,$$

-No waiting after operating

$$o_{iv,t-1} \ge \sum_{j\in N\cup\{d(v)\}} x_{ijvt}, \quad v\in V, i\in N, t\in T,$$

-Must operate before sailing

$$\sum_{v\in V} o_{ivt} \le B_{it}, \quad i\in N, t\in T,$$

-Berth capacity

$$0 \le q_{ivt} \le Q_v o_{ivt}, \quad v\in V, i\in N, t\in T,$$

-UB on(un)loading

$$s_{i,t-1} + \sum_{v\in V} q_{ivt} = D_{it} + s_{it}, \quad i\in N^D, t\in T,$$

$$s_{i,t-1} + P_{it} = \sum_{v\in V} q_{ivt} + s_{it}, \quad i\in N^P, t\in T,$$

$$\underline{S}_{it} \le s_{it} \le \overline{S}_{it}, \quad i\in N, t\in T,$$

$$s_{i0} = S^0_i, \quad i\in N,$$

-Port inventory management

$$l_{v,t-1} + \sum_{i\in N^P} q_{ivt} - \sum_{i\in N^D} q_{ivt} - l_{vt} = 0, \quad v\in V, t\in T,$$

$$0 \le l_{vt} \le K_v, \quad v\in V, t\in T,$$

$$l_{v0} = L^0_v, \quad v\in V,$$

- Ship load management

$$x_{ijvt} \in \{0,1\}, \quad v\in V, i\in N\cup\{o(v)\}, j\in N\cup\{d(v)\}, t\in T,$$

$$o_{ivt}, w_{ivt} \in \{0,1\}, \quad v\in V, i\in N, t\in T.$$

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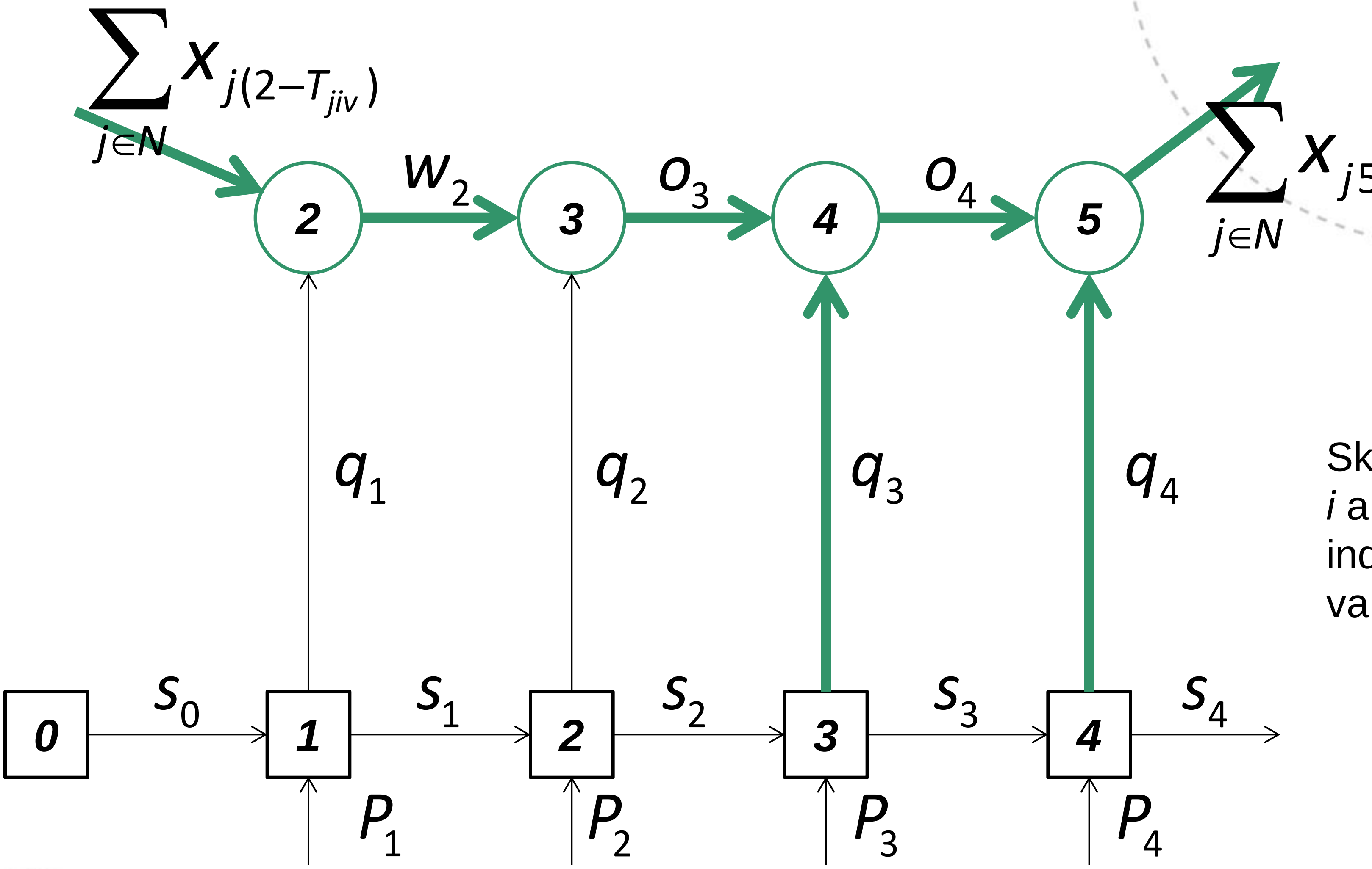
Loading operation at port *i* with ship *v*



Skipped *i* and *v* indices on variables

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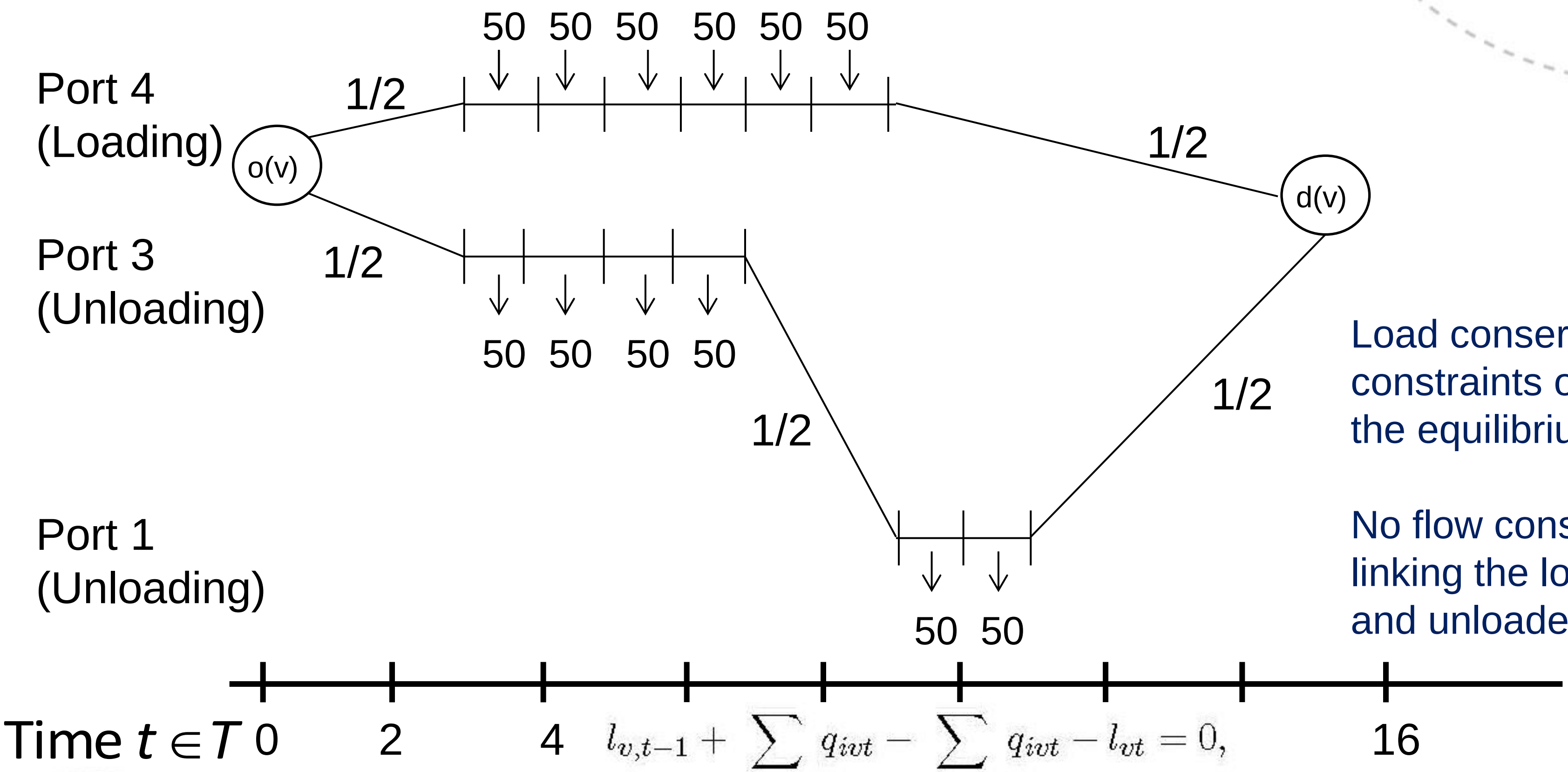
A good model?

Instance	(|N|,|V|)	OPT	LR	GAP
A	(3,2)	137.4	22.3	83.8
B	(4,2)	370.6	32.0	91.4
C	(4,2)	413.5	44.7	89.2
D	(5,2)	357.9	53.6	85.0
E	(5,2)	355.5	52.3	85.3
F	(4,3)	504.9	105.2	79.2
G	(6,5)	747.9	213.6	71.4
Average				**83.6**

OPT – Optimal solution value
LR – Value of the linear relaxation
GAP – Integrality gap = 100%*(OPT-LR)/OPT

A good model?

- The linear relaxation allows fractional solutions such as this one:



$$l_{v,t-1} + \sum_{i \in N^P} q_{ivt} - \sum_{i \in N^D} q_{ivt} - l_{vt} = 0,$$

Load conservation constraints only ensure the equilibrium onboard

No flow constraints linking the loaded and unloaded quantities

Idea

- Assign a flow to the quantity on board the ship for each link on the ship path

$$l_{vt} \quad \rightarrow \quad f_{ijvt}$$

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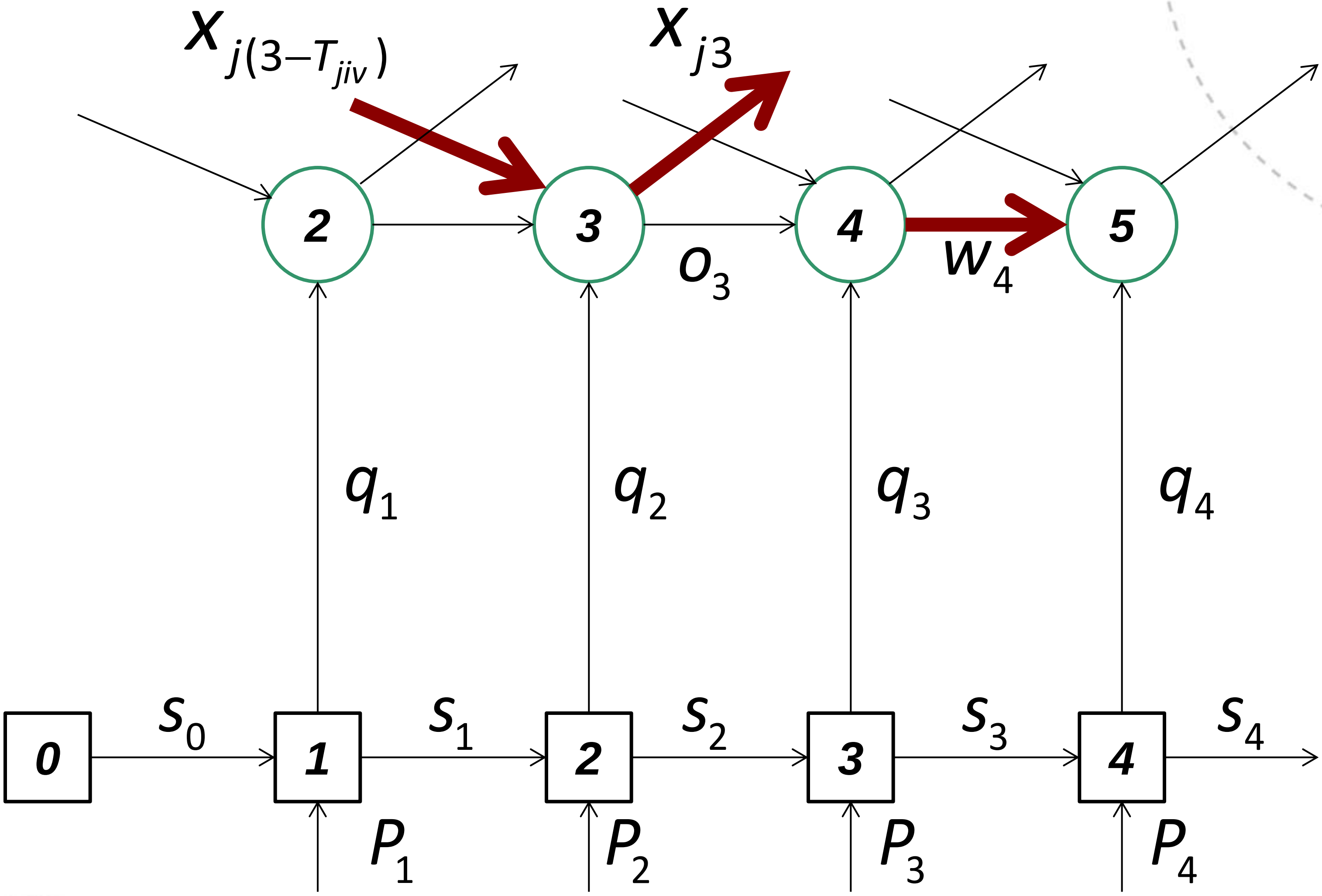
Not allowed sequences of actions



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Fixed Charge Network Flow (FCNF) model

- To prevent not allowed sequences, introduce new operation binary variables:

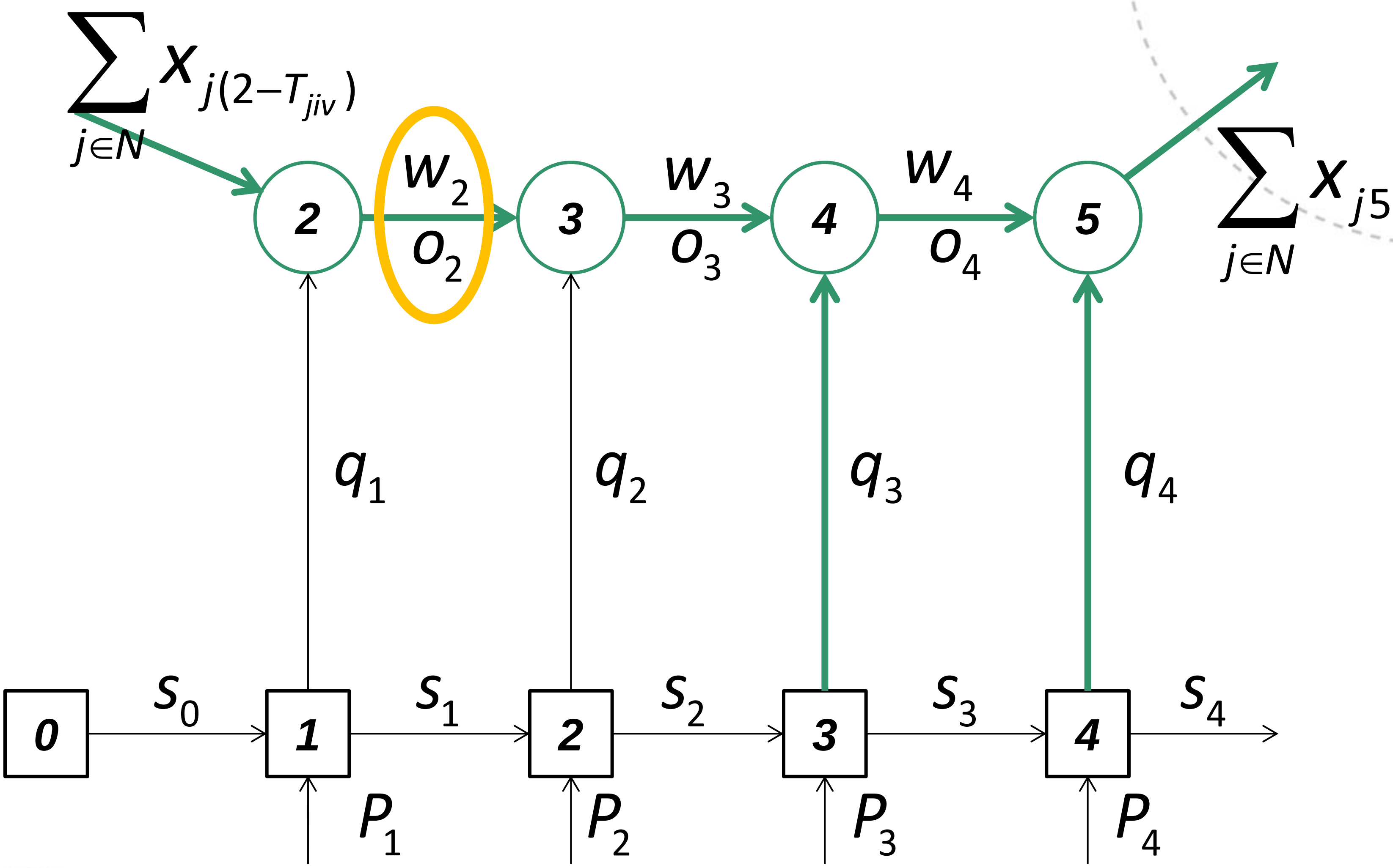
o_{ivt}^{A} = 1 if ship *v* starts to operate at port *i* in period *t*/ =0 otherwise

o_{ivt}^{B} = 1 if ship *v* continues operating at port *i* in period *t*/ =0 otherwise

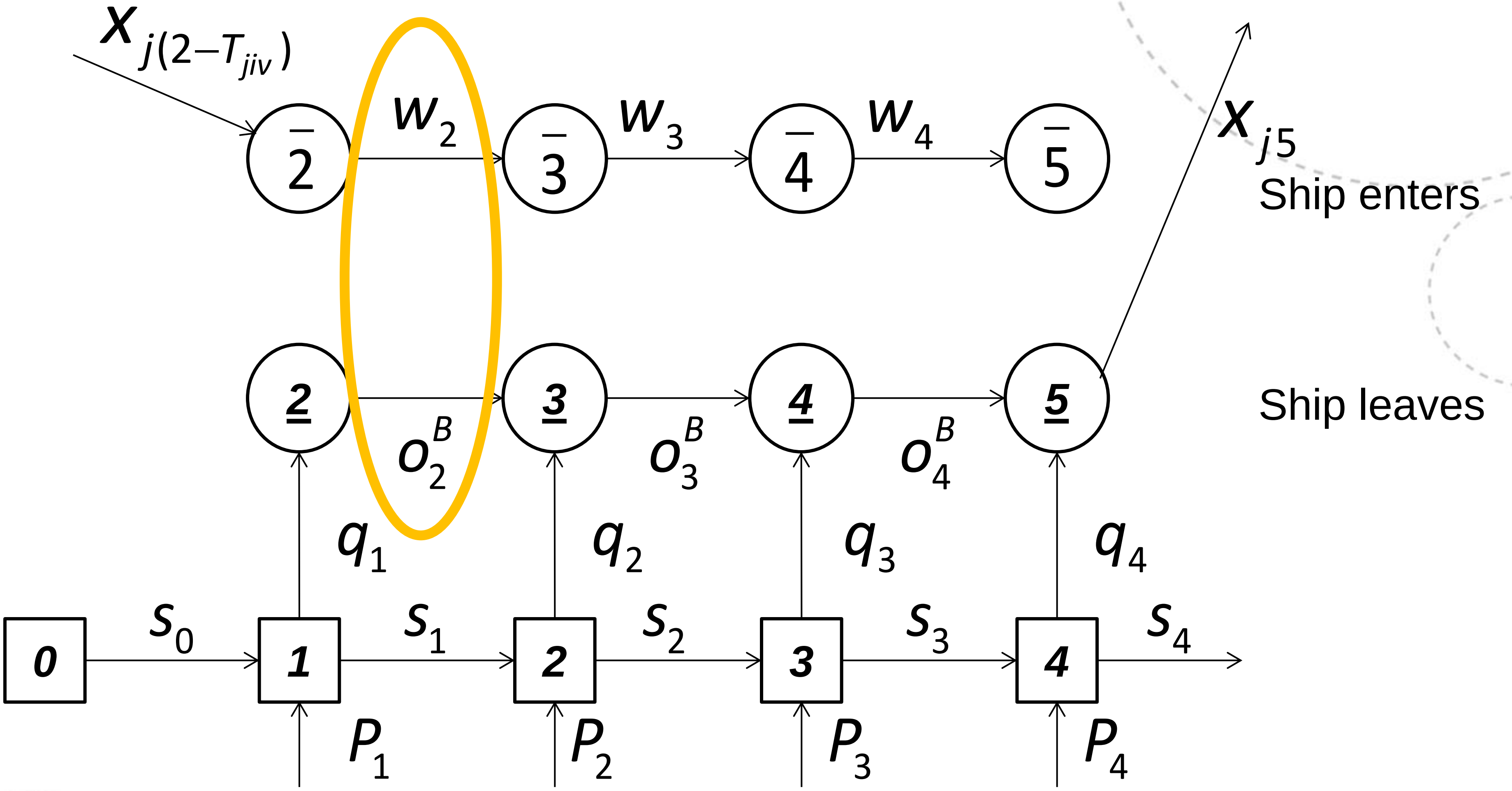
- Hence,

$$o_{ivt} = o_{ivt}^{A} + o_{ivt}^{B}$$

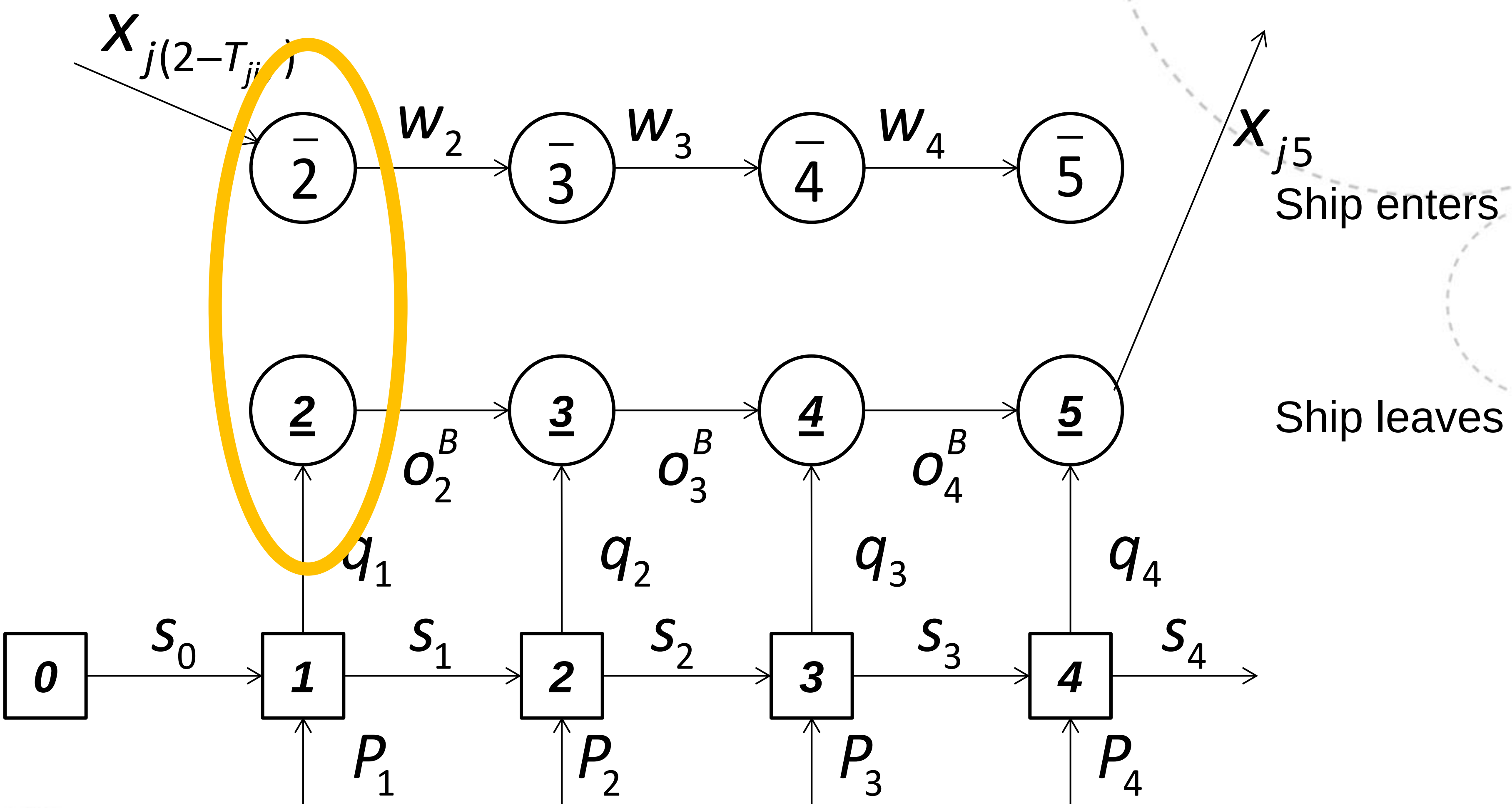
Original formulation: Loading - Revisited



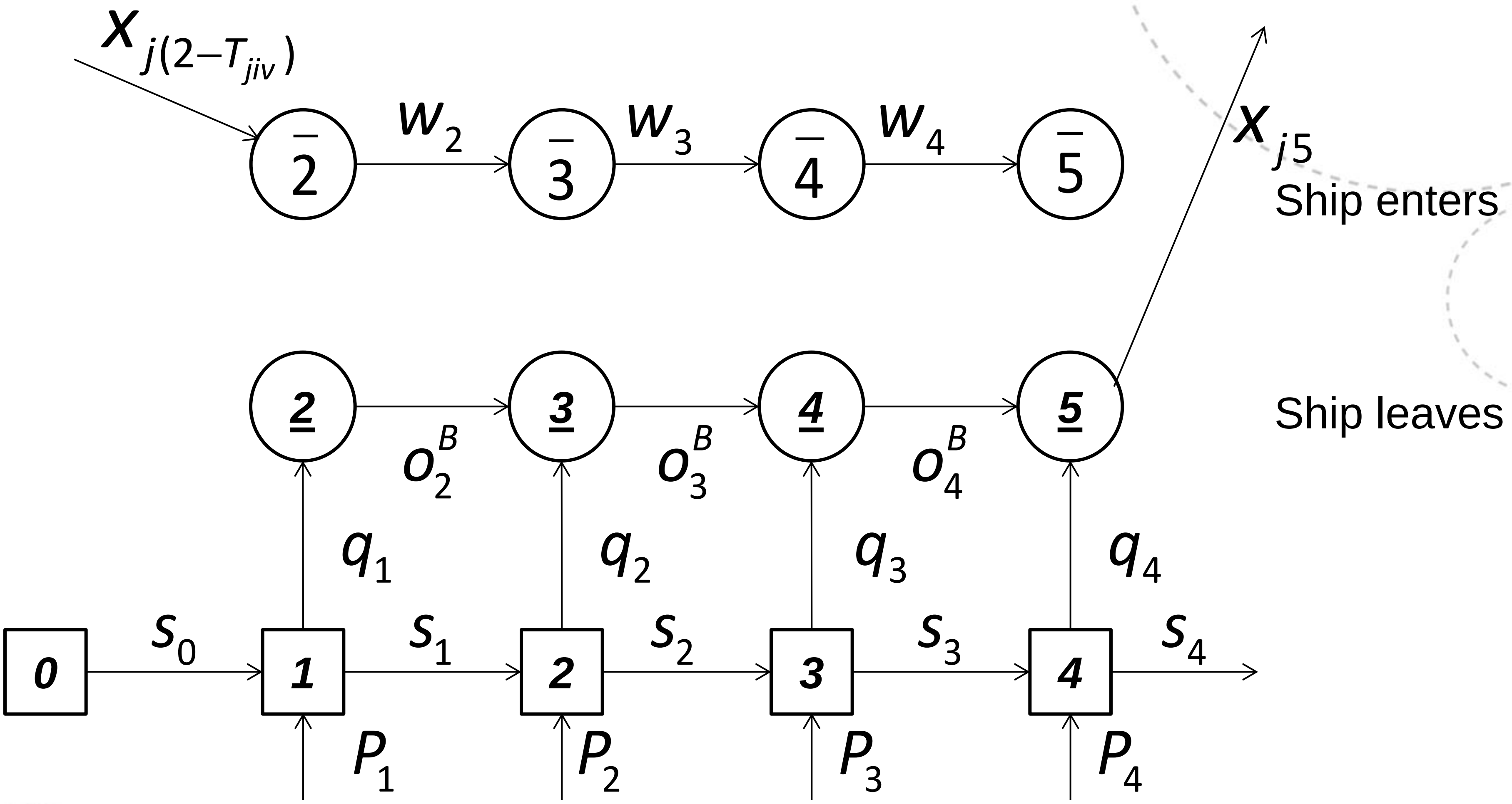
Extended network: Loading operation



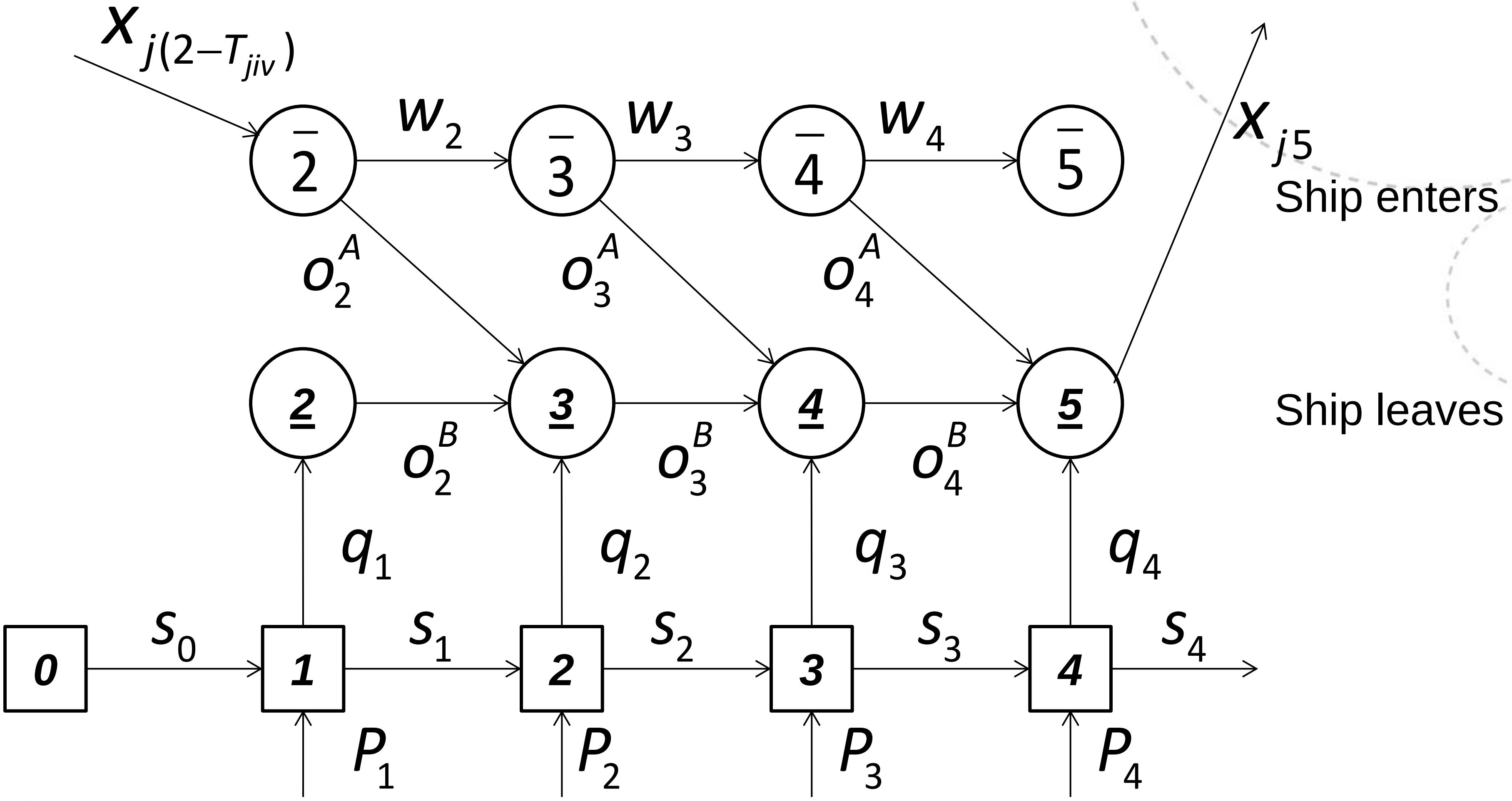
Extended network: Loading operation



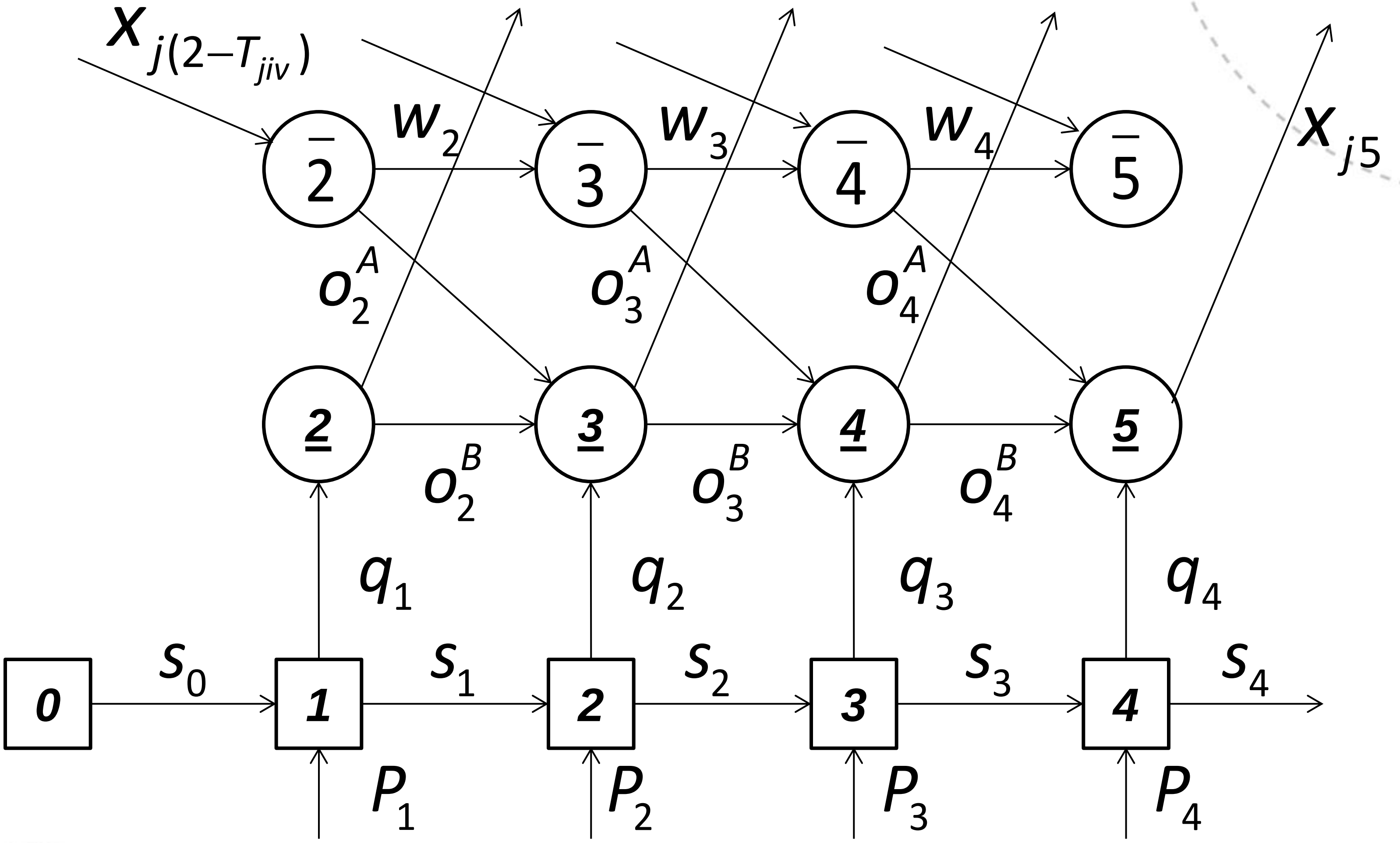
Extended network: Loading operation



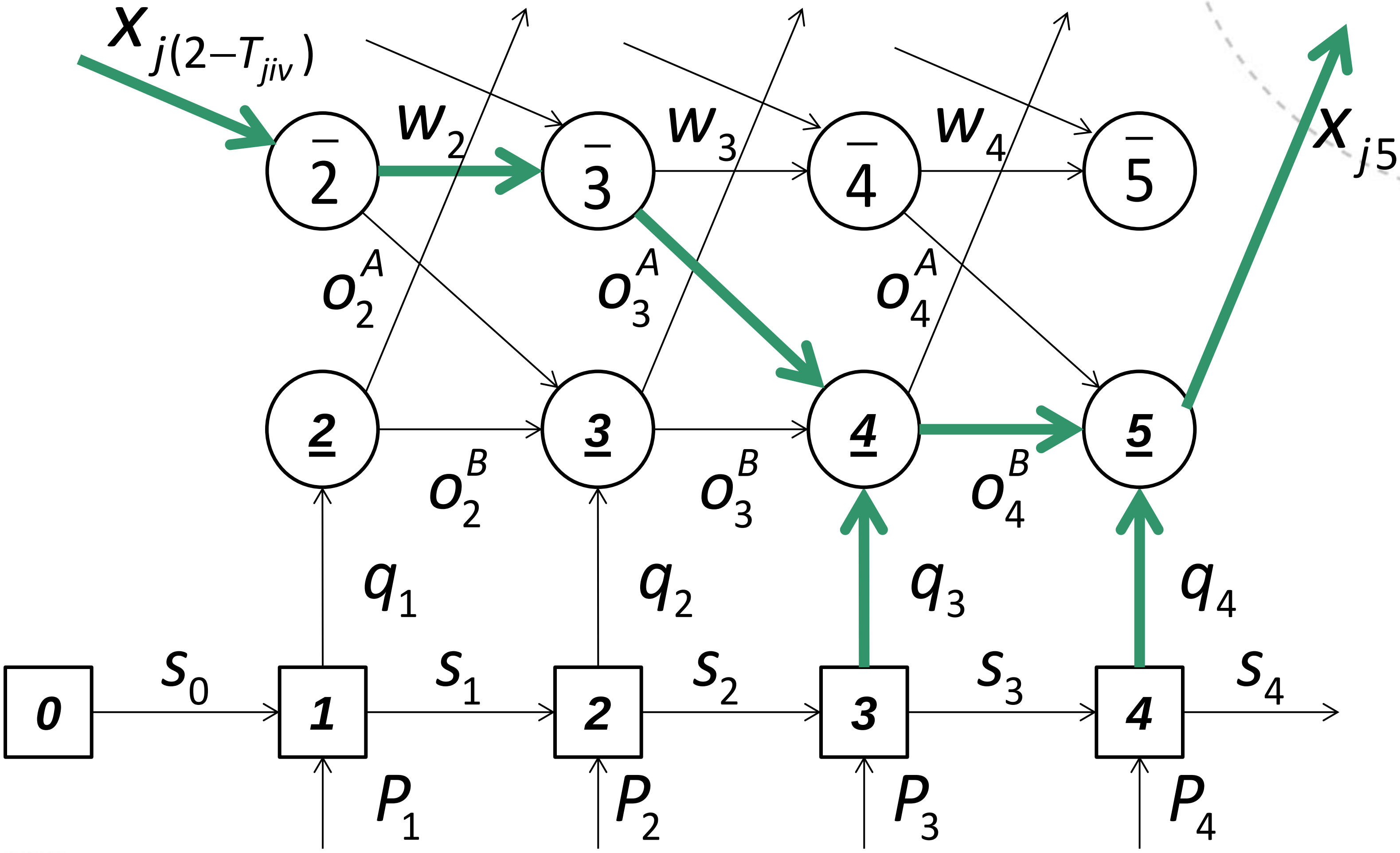
Extended network: Loading operation



Extended network: Loading operation



Extended network: Loading operation



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Ship flow conservation

Original:

$$\sum_{j \in N \cup \{o(v)\}} x_{jiv,t-T_{jiv}} + w_{iv,t-1} + o_{iv,t-1} = \sum_{j \in N \cup \{d(v)\}} x_{ijvt} + w_{ivt} + o_{ivt}, \quad v \in V, i \in N, t \in T,$$

$$o_{iv,t-1} \leq \sum_{j \in N \cup \{d(v)\}} x_{ijvt} + o_{ivt}, \quad v \in V, i \in N, t \in T,$$

$$o_{iv,t-1} \geq \sum_{j \in N \cup \{d(v)\}} x_{ijvt}, \quad v \in V, i \in N, t \in T,$$

FCNF:

$$\sum_{j \in N \cup \{o(v)\}} x_{jiv,t-T_{jiv}} + w_{iv,t-1} = w_{ivt} + o^A_{ivt} \qquad v \in V, i \in N, t \in T,$$

First layer

$$o^A_{iv,t-1} + o^B_{iv,t-1} = o^B_{ivt} + \sum_{j \in N \cup \{d(v)\}} x_{ijvt}, \qquad v \in V, i \in N, t \in T,$$

Second layer

$$o^A_{ivt} + o^B_{ivt} = o_{ivt}, \qquad v \in V, i \in N, t \in T,$$

$$o^A_{ivt}, o^B_{ivt} \in \{0, 1\}, \qquad v \in V, i \in N, t \in T,$$

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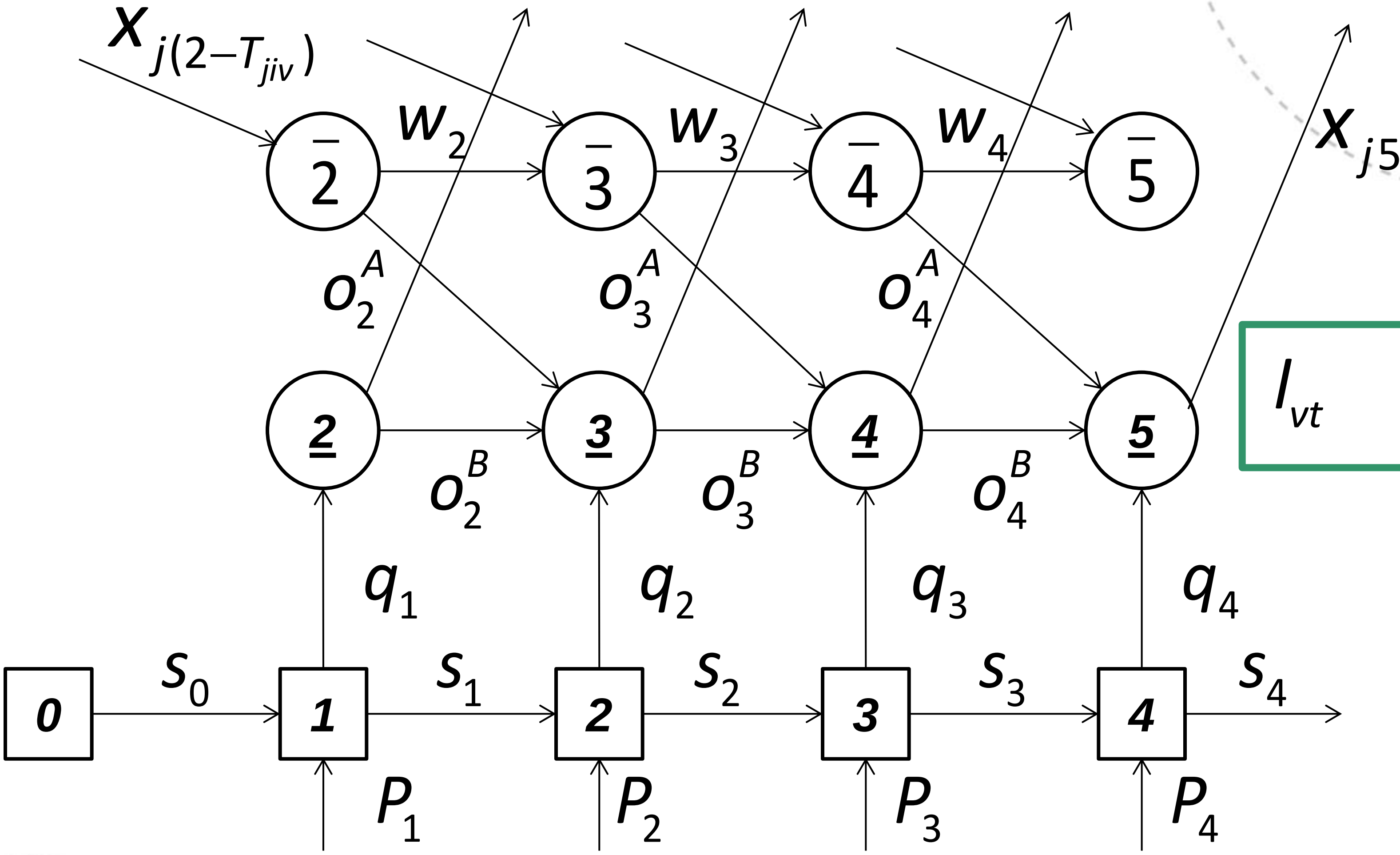
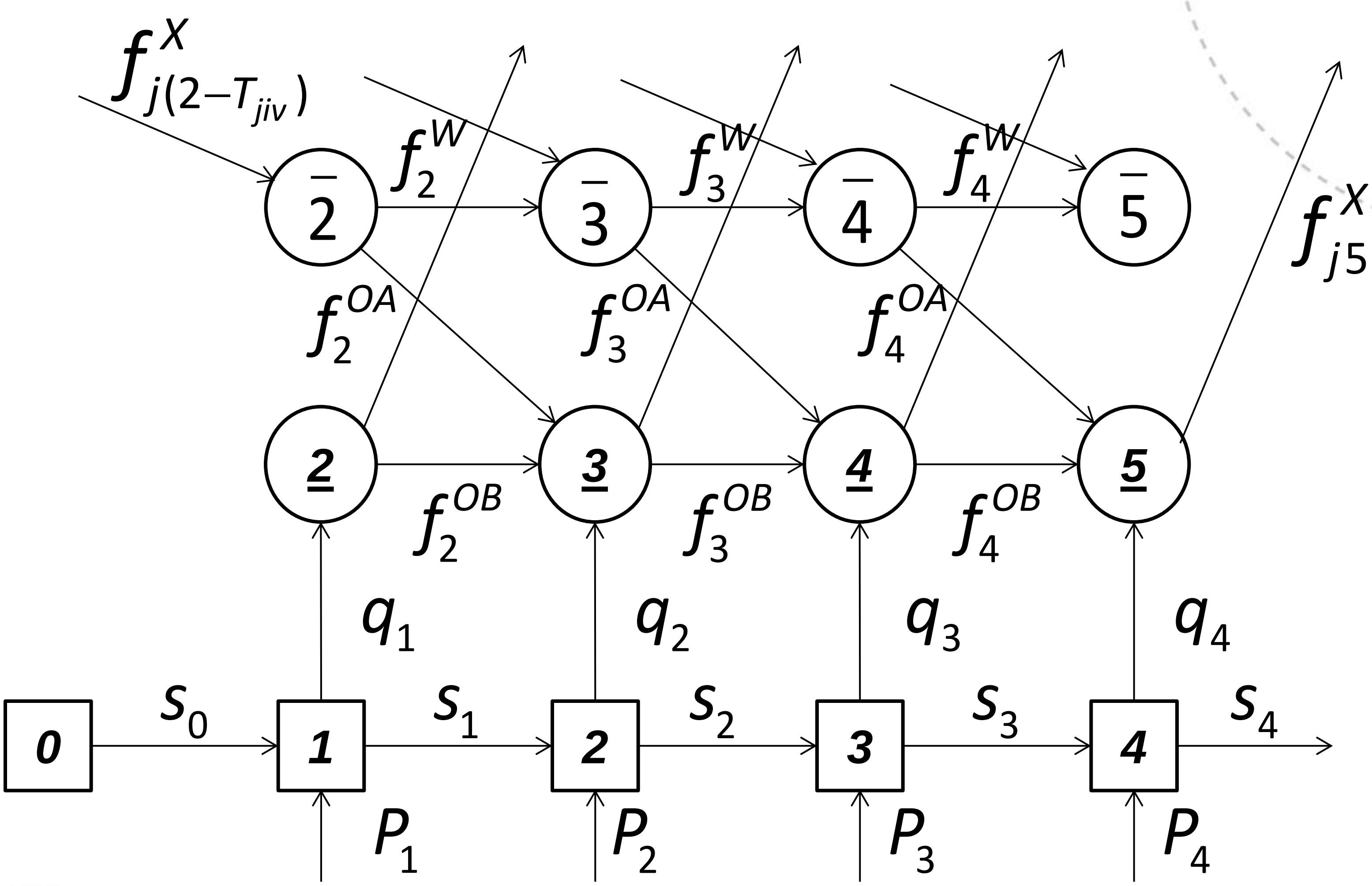
Extended network: Revisited



$l_{vt} \rightarrow f_{ijvt}$

Extended network with flow variables



Ship Load Management

Original:

$$l_{v,t-1} + \sum_{i \in N^P} q_{ivt} - \sum_{i \in N^D} q_{ivt} - l_{vt} = 0, \qquad v \in V, t \in T,$$

$$0 \leq l_{vt} \leq K_v, \qquad v \in V, t \in T,$$

$$l_{v0} = L_v^0, \qquad v \in V,$$

FCNF:

First layer

$$\sum_{j \in N \cup \{o(v)\}} f^X_{jiv,t-T_{jiv}} + f^W_{iv,t-1} = f^W_{ivt} + f^{OA}_{ivt} \qquad v \in V, i \in N, t \in T,$$

Second layer

$$f^{OA}_{iv,t-1} + f^{OB}_{iv,t-1} + q_{iv,t-1} = f^{OB}_{ivt} + \sum_{j \in N \cup \{d(v)\}} f^X_{ijvt}, \qquad v \in V, i \in N^P \cup \{o(v)\}, t \in T,$$

$$f^{OA}_{iv,t-1} + f^{OB}_{iv,t-1} - q_{iv,t-1} = f^{OB}_{ivt} + \sum_{j \in N \cup \{d(v)\}} f^X_{ijvt}, \qquad v \in V, i \in N^D \cup \{o(v)\}, t \in T,$$

$$f^X_{o(v)jvt} = L_v^0 x_{o(v)jvt} \qquad \forall v \in V, j \in N \cup \{d(v)\},$$

$$0 \leq f^X_{ijvt} \leq K_v x_{ijvt} \qquad v \in V, i \in N \cup \{o(v)\}, j \in N \cup \{d(v)\}, t \in T,$$

$$0 \leq f^{OA}_{ivt} \leq K_v o^A_{ivt} \qquad v \in V, i \in N, t \in T,$$

$$0 \leq f^{OB}_{ivt} \leq K_v o^B_{ivt} \qquad v \in V, i \in N, t \in T,$$

$$0 \leq q_{ivt} \leq Q_v o_{ivt} \qquad v \in V, i \in N, t \in T,$$

$$0 \leq f^W_{ivt} \leq K_v w_{ivt} \qquad v \in V, i \in N, t \in T.$$

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FCNF model

- A pure fixed charge network flow (FCNF) model with side constraints
  - Introducing flow load variables
  - Splitting the operation variables into start-up and continuing operation variables
  - Changing the ship load management and flow conservation constraints

Comparing original and FCNF models

Instance	(|N|,|V|)	Original: Gap	Original: Time	FCNF: Gap	FCNF: Time
A	(3,2)	83.8	1	49.3	7
B	(4,2)	91.4	11	28.9	6
C	(4,2)	89.2	1700	43.0	*
D	(5,2)	85.0	117	43.0	94
E	(5,2)	85.3	268	42.0	136
F	(4,3)	79.2	*	30.6	*
G	(6,5)	71.4	*	17.3	*
Average		**83.6**		**36.3**	

GAP – 100%*(OPT-LR)/OPT

* Optimal solution not found within 3 hours

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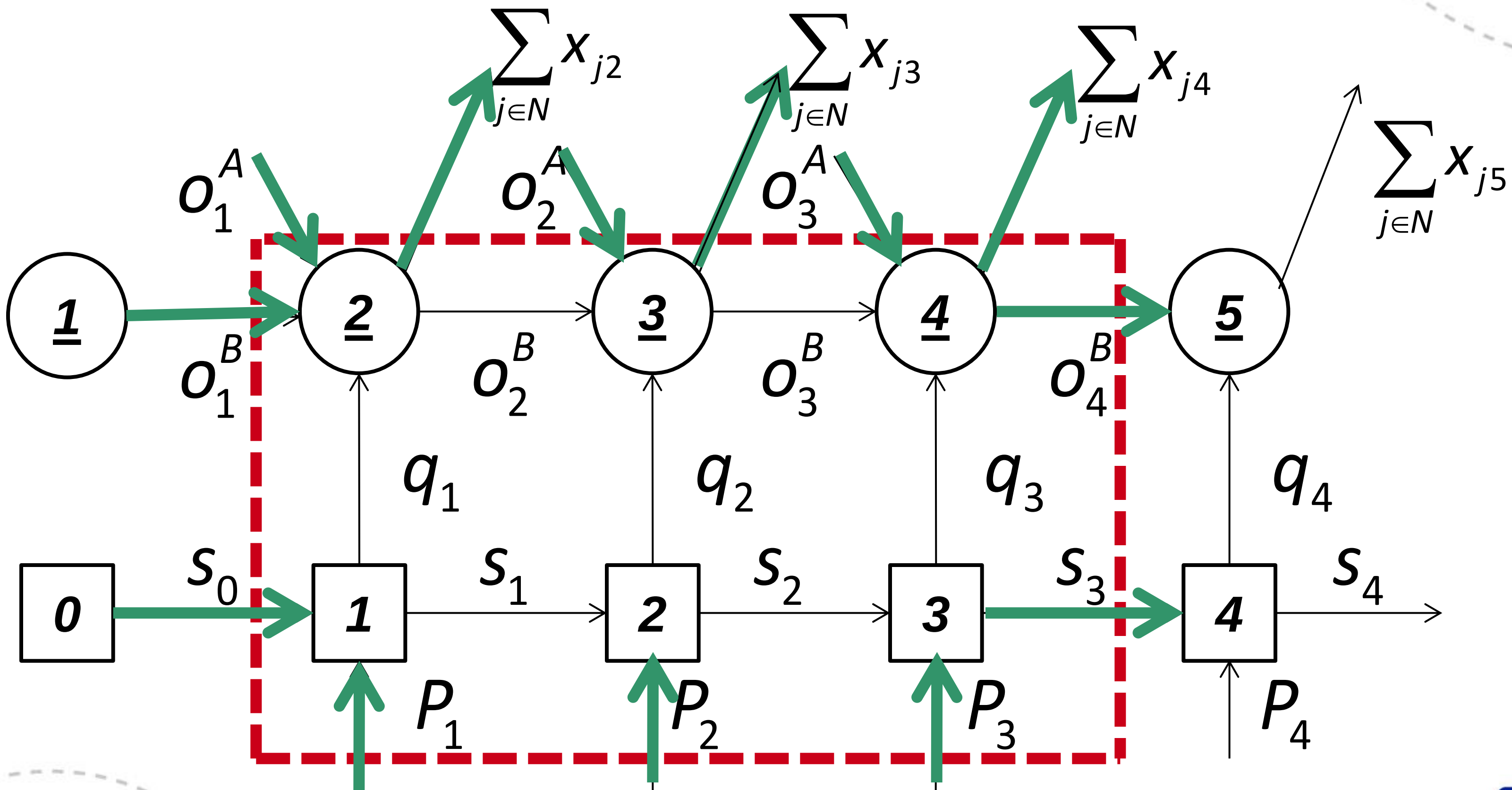
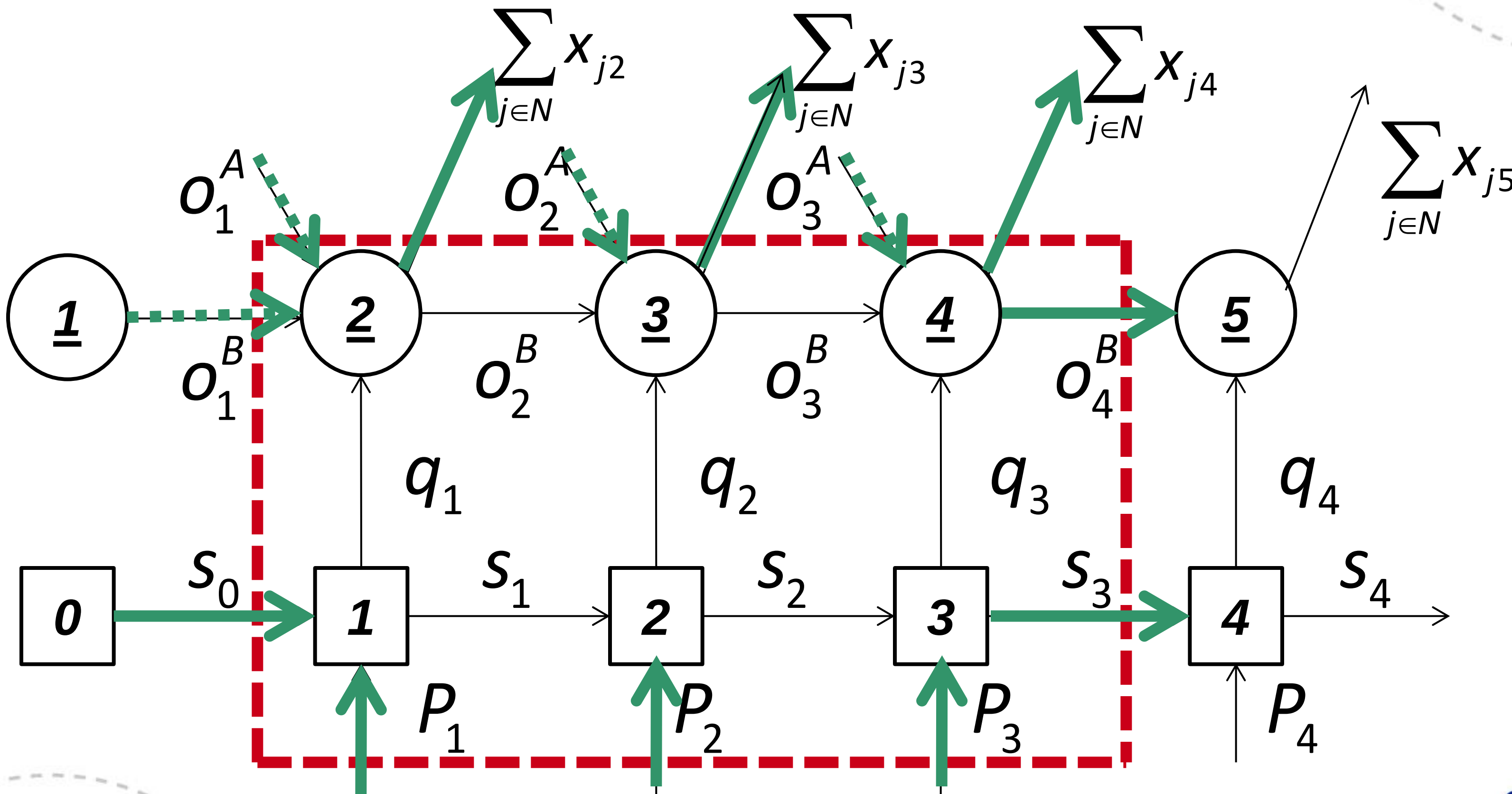
Mixed integer routing inequalities: Loading port



Mixed integer routing inequalities: Loading port

$$\boxed{s_3 + Qo_4^B + \sum_{j\in N} Kx_{j2} + \sum_{j\in N} Kx_{j3} + \sum_{j\in N} Kx_{j4} \geq P_1 + P_2 + P_3 + \underline{S}_0}$$

Basis for a MIR inequality



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MIR and knapsack inequalities

Basis for MIR inequalities:

$$\frac{s_{ik}}{Q} + \sum_{v \in V} \left(\sum_{t \in \mathrm{T}_v^+} \frac{K_v}{Q} o^B_{iv,t+1} + \sum_{j \in N \cup \{d(v)\}} \sum_{t \in \mathrm{T}_v} \frac{K_v}{Q} x_{ijv,t+1} + \sum_{t \in \mathrm{T} \setminus \mathrm{T}_v} \frac{Q_v}{Q} o_{ivt} \right) \geq \left(\sum_{t \in \mathrm{T}} P_{it} + \underline{S}_{i,\ell-1} \right) / Q$$

Knapsack inequalities:

$$\sum_{v \in V} \left(\sum_{t \in \mathrm{T}_v^+} \left\lceil \frac{K_v}{Q} \right\rceil o^B_{iv,t+1} + \sum_{j \in N \cup \{d(v)\}} \sum_{t \in \mathrm{T}_v} \left\lceil \frac{K_v}{Q} \right\rceil x_{ijv,t+1} + \sum_{t \in \mathrm{T} \setminus \mathrm{T}_v} \left\lceil \frac{Q_v}{Q} \right\rceil o_{ivt} \right) \geq \left\lceil \frac{\sum_{t \in \mathrm{T}} P_{it} + \underline{S}_{i,\ell-1} - \overline{S}_{ik}}{Q} \right\rceil$$

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Lot-sizing relaxations

- Single-item lot-sizing with constant capacity (LSCC)
  - Production q_t of a single product with demand D_t in period t
  - Production capacity $\bar{Q}$
  - Inventory s_t of the product at the end of period t
  - A binary set-up variable taking the value $o_t = 1$ if there is production in the period (q_t>0) (Alternatively, o_t as the number of batches of maximum size Q that is required to produce q_t)

- Relationship between a LSCC set and MIRP inventory management set
  - Production in LSCC ->Unloading in MIRP (similar reasoning for loading)
  - Number of batches ->Number of ships operating in a port at the same time

LSCC and MIRP inventory management set

LSCC set (1)

$$\begin{aligned} \tilde{s}_{i,t-1} + \tilde{q}_{it} &= \tilde{D}_{it} + \tilde{s}_{it}, && t \in T, \\ \tilde{q}_{it} &\leq \overline{Q}\tilde{o}_{it}, && t \in T, \\ \tilde{q}_{it}, \tilde{s}_{it} &\geq 0, && t \in T, \\ \tilde{o}_{it} &\in Z^1_+ && t \in T. \end{aligned}$$

Inventory management set(2)

$$\begin{aligned} s_{i,t-1} + \sum_{v \in V} q_{ivt} &= D_{it} + s_{it}, && t \in T, \\ 0 \leq q_{ivt} &\leq Q_v o_{ivt}, && v \in V, t \in T, \\ \sum_{v \in V} o_{ivt} &\leq B_{it}, && t \in T, \\ \underline{S}_{it} \leq s_{it} &\leq \overline{S}_{it}, && t \in T, \\ s_{i0} &= S^0_i, && \\ o_{ivt} &\in \{0,1\}, && v \in V, t \in T. \end{aligned}$$

- (2) can easily be transformed to (1)

- Several known valid inequalities for LSCC

- A relaxation of the LSCC is the Wagner-Whitin constant capacity lot-sizing set (WWCC), for which polynomial size extended formulations exist

Lot-sizing with Start-Up Relaxations

- Important extension of LSCC:
  - include start-up costs with the first period of an interval of set-ups (LSCCS)
- Start-up: Corresponds to the first period a ship operates in a port
- In the FCNF model we have o_{ivt}^{A} as a start-up variable, but not in the original model
- For this problem several known valid inequalities exist
- Valid inequalities for the LSCCS are used to derive valid inequalities for our FCNF model

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Computational study

- (V,B) – (Valid inequalities (C), Branching priority on variables (Sx,SoA))
- Sx – # of times ship *v* visits port *i*   /   SoA – # of start-ups of ship v at port *i*

- Computing times in seconds:

Instance	(|N|,|V|)	Original			FCNF		
		(-,-)	(C,-)	(C,Sx)	(-,-)	(C,-)	(C,SoA)
A	(3,2)	1.3	0.2	0.2	7	0.5	0.5
B	(4,2)	11	6	3	6	5	4
C	(4,2)	1700	310	105	*	147	19
D	(5,2)	117	5	5	94	9	10
E	(5,2)	268	10	4	136	21	16
F	(4,3)	*	*	1754	*	317	53
G	(6,5)	*	*	3226	*	111	54

- Xpress Optimizer version 21.01.00 with Xpress Mosel Version 3.2.0
- Computer with processor Intel Core 2 Duo, CPU 2.2 GHz, with 4 GB of RAM

Average integrality gaps

GAP – 100%*(OPT-LR)/OPT

Original				FCNF			
L	X	C	C,X	L	X	C	C,X
83.6	57.5	14.4	10.1	36.3	8.8	11.4	6.4

L – No cuts or Lot-sizing reformulations
X – With Xpress cuts
CR – With all valid inequalities (knapsack, MIR, Lot-sizing) generated and reformulations
CR,X – With both Xpress cuts, valid inequalities and reformulations

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Gaps closed (%) with cuts and reformulations

Instance	Original				FCNF				
	X	K	W	K,W	X	K	W	D	K,W,D
A	80.1	100	24.2	**100**	53.0	100	100	100	**100**
B	21.8	78.6	48.4	**91.4**	**72.2**	42.4	44.5	16.3	46.0
C	16.0	79.6	51.0	**89.2**	54.5	64.0	56.6	10.6	**64.3**
D	43.4	75.5	46.3	**85.1**	52.6	58.8	52.9	10.9	**62.1**
E	25.8	74.5	43.9	**85.4**	54.8	56.0	52.2	9.2	**59.0**
F	11.3	**81.3**	23.5	79.2	**64.3**	60.5	58.8	7.1	58.7
G	19.1	**92.0**	43.0	71.6	80.0	79.3	66.0	16.7	**80.1**

X – Xpress cuts
K – Knapsack cuts (and MIR)
W – Wagner Whitin constant capacity lot-sizing reformulation
D – Inequalities for lot-sizing with start-up relaxations

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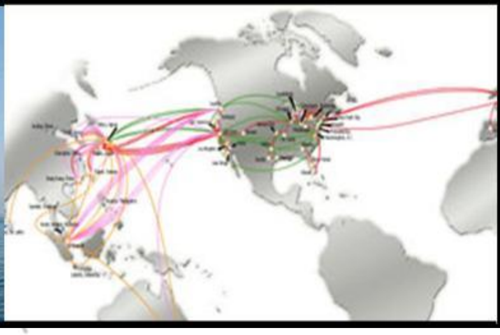
Concluding remarks

- Two discrete time formulations are introduced for a MIRP with time varying production/consumption rates
  - Original formulation
  - FCNF formulation with side constraints
- The formulations are strengthened using valid inequalities from mixed integer sets that arise as relaxations of the formulations + lot-sizing reformulations
- FCNF formulation provides far better bounds than the original
- Combining valid inequalities and cuts from Xpress reduces the integrality gaps of both formulations
- Valid inequalities and branching strategy based on start-up variables made it possible to solve large instances to optimality (results not shown)

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