GERAD

A matheuristic for the liner shipping network design problem with transit times

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DTU Management Engineering & GERAD

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Liner shipping networks


Urban transit network

vs.

Liner shipping network

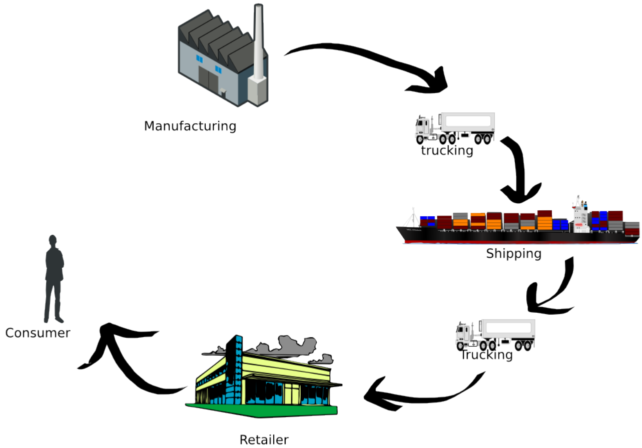

Commuters

vs.


Containers

Why is liner shipping important?

Integrated part of global supply chains

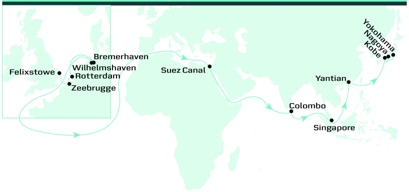


High value goods

Liner shipping network design problem (LSNDP)

Constructing a set of cyclic routes with a fixed frequency

Asia - Europe (AE1) - Eastbound



Port	Arrives	Departs	Transit
Felixstowe, United Kingdom	TUE	WED	--
Zeebrugge, Belgium	THU	THU	1
Rotterdam, Netherlands	SAT	SAT	3
Bremerhaven, Germany	SUN	MON	5
Wilhelmshaven, Germany	MON	TUE	6
Suez Canal, Egypt	TUE	WED	13
Colombo, Sri Lanka	THU	THU	22
Singapore, Singapore	WED	THU	28
Yantian, China	WED	THU	35
Kobe, Japan	TUE	TUE	41
Nagoya, Japan	WED	THU	42
Yokohama, Japan	FRI	FRI	44

A transportation network with colossal costs

Cost of the AE1 schedule (weekly)

- 10 vessels
- Weekly cost (Million $)
- Vessel sizes in 20 foot container units (TEU - Twenty Foot Equivalent)

	8.400 TEU (10 vessels)	15.000 TEU (10 vessels)
Port Call Cost	$0.47	$0.75
Canal Cost	$1.26	$2.07
Vessel Cost	$2.45	$3.85
Fuel Cost	$3.15	$4.86
Total Cost	$7.35	$11.54

Calculations are based on distances and cost of *LINER-LIB 2012* assuming a weekly frequency.

Recent literature on LSNDP

- Agarwal & Ergun (2008)
- Alvarez (2009)
- Blander Reinhardt & Pisinger (2012)
- Brouer et al. (2014)
- Brouer, Desaulniers, Pisinger (2014)

General trend

- Complex mathematical models
- Two tiers:
  1. Route construction
  2. Multi commodity flow problem
- Solved heuristically except for Blander Reinhardt & Pisinger (2012)

What about the transit times?

Effect of imposing transit time

Instance	Trans% (MCF)	Trans% (MCF-TT)
Baltic	92.1	92.1
WAF	94.9	74.5
Med.	95.3	71.2
Pacific	91.1	58.8
WorldSmall	91.3	60.4
AsiaEurope	91.2	76.1

- The route network is optimized for cost/capacity utilization
- The percentage of cargo flowed differs significantly
  1. if solving the original multi commodity flow problem (MCF)
  2. OR the MCF with transit time restrictions (MCF-TT)

Towards competitive liner shipping network design models

Transit time is a competition factor

- Maximizing revenue means optimizing the capacity utilization
- Results in longer port call sequences
- Transit time likely to increase
- Transit time is crucial for the commercial value of a product

Literature on LSNDP-TT limited

- Wang & Meng 2014 (LSNDPD) [8]
- Non-linear, non-convex MIP model
- Coupling a service route with a cargo delivery pattern
- Shortcoming: No support for transhipments

- Karsten et. al. 2014 on time constrained multi commodity flow problem (MCF-TT) [7]
- Working paper

A reference model for LSNDP

$$(VRD) \quad \text{minimize} \quad Z =$$

$$\sum_{r\in\mathbf{R}} f^{v_r} Y^r + \sum_{v\in\mathbf{V}} \tilde{f}^v \left(z^v - \sum_{r\in\mathbf{R}:v_r=v} Y^r \right) + \qquad (1a)$$

$$\sum_{r\in\mathbf{R}} m_r Y^r \sum_{(i,j)\in\mathbf{E}_r} \left(eh^{v_r} p_j^{v_r} + eg^{v_r s_r} l_{ij}^{v_r} + d_j^{v_r} + a_{ij}^{v_r} \right) + \qquad (1b)$$

$$\sum_{(o,d)\in\mathbf{G}} \left(\tilde{q}_{od} O_{od} - q_{od} \sum_{r\in\mathbf{R}} V_{od}^r \right) + \qquad (1c)$$

$$\sum_{r\in\mathbf{R}} \sum_{(h,i,j)\in\Omega_r} \left(u_i \left(W_{(hi)}^r + \sum_{\substack{d\in\mathbf{P}\\ d\neq i}} V_{id}^r \right) + t_i \sum_{\substack{d\in\mathbf{P}\\ d\neq i}} \sum_{\substack{s\in\mathbf{R}\\ s\neq r}} U_{(hi)d}^{rs} \right) \qquad (1d)$$

A reference model for LSNDP (cont.)

$$X^r_{(hi)d} + V^r_{id} + \sum_{\substack{s \in \mathbf{R}: \\ (k,i,l) \in \Omega_s \\ s \neq r}} U^{sr}_{(ki)d} = X^r_{(ij)d} + \sum_{\substack{s \in \mathbf{R} \\ s \neq r}} U^{rs}_{(hi)d}$$

$$r \in \mathbf{R}, (h,i,j) \in \Omega_r, d \in \mathbf{P} i \neq d h \neq d \qquad (2)$$

$$X^r_{(ij)j} = W^r_{(ij)} \quad (i,j) \in \mathbf{E}_r \quad r \in \mathbf{R} \qquad (3)$$

$$O_{od} + \sum_{r \in \mathbf{R}} V^r_{od} = k_{od} \quad o,d \in \mathbf{G} \qquad (4)$$

$$\sum_{d \in \mathbf{P}} X^r_{ijd} \leq c^{v_r} \cdot m_r \cdot Y^r \quad r \in \mathbf{R} \quad (i,j) \in \mathbf{E}_r \qquad (5)$$

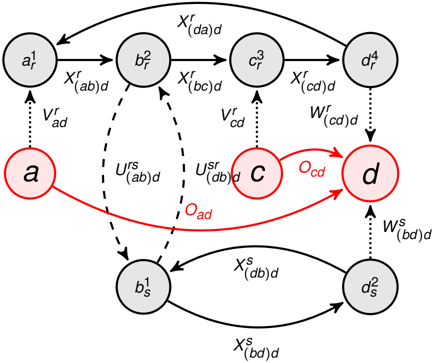
$$\sum_{r \in \mathbf{R}: v_r = v} Y^r \leq z^v \quad v \in \mathbf{V} \qquad (6)$$

$$(7)$$

$$X^r_{(ij)d}, U^{rs}_{(ij)d}, W^r_{id} \in \mathbb{R}^+ \quad r,s \in \mathbf{R}, r \neq s \quad (i,j) \in \mathbf{E}_r \quad d \in \mathbf{P} \qquad (8)$$

$$O_{od}, V^r_{od} \in \mathbb{R}^+ \quad r \in \mathbf{R} \quad o,d \in \mathbf{G} \qquad (9)$$

$$Y^r \in \mathbb{Z}^+ \quad r \in \mathbf{R} \qquad (10)$$



Arc flow model of the multicommodity flow problem

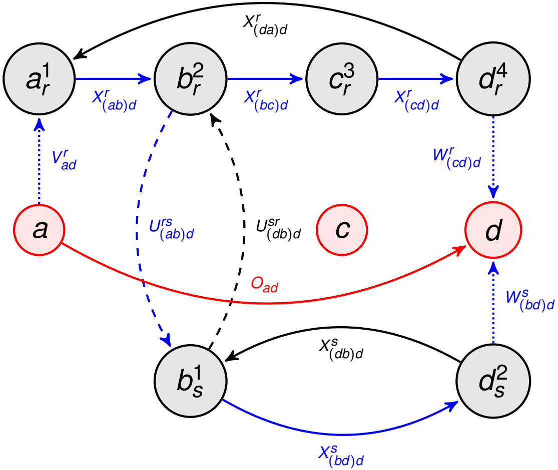


Figure: The flow for commodity (a,d) (blue arcs) constitute a tree of flow

Path flow formulation of the multicommodity flow problem

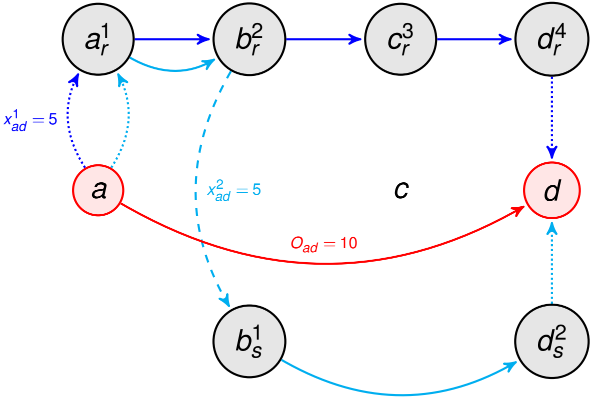


Figure: The flow tree for commodity (a,d) may be seen as two distinct paths

$$(VRD) \quad \text{minimize} \quad Z =$$

$$\sum_{r\in\mathbf{R}} f^{v_r} Y^r + \sum_{v\in\mathbf{V}} \tilde{f}^v \left(z^v - \sum_{r\in\mathbf{R}:v_r=v} Y^r \right) + \qquad (11a)$$

$$\sum_{r\in\mathbf{R}} m_r Y^r \sum_{(i,j)\in\mathbf{E}_r} \left(eh^{v_r} p_j^{v_r} + eg^{v_r s_r} l_{ij}^{v_r} + d_j^{v_r} + a_{ij}^{v_r} \right) + \qquad (11b)$$

$$\sum_{g\in\mathbf{G}} \left(\tilde{q}_g O_g - q_g \sum_{p\in P_g} x^p \right) + \qquad (11c)$$

$$\sum_{g\in\mathbf{G}} \sum_{p\in P_g} c_p x^p \qquad (11d)$$

$$O_g + \sum_{p \in P_g} x^p = k_g \quad g \in \mathbf{G} \tag{12}$$

$$\sum_{g \in \mathbf{G}} \sum_{p \in P_g} \delta_{ij}^p x^p \leq c^{v_r} \cdot m_r \cdot Y^r \quad r \in \mathbf{R} \quad (i,j) \in \mathbf{E}_r \tag{13}$$

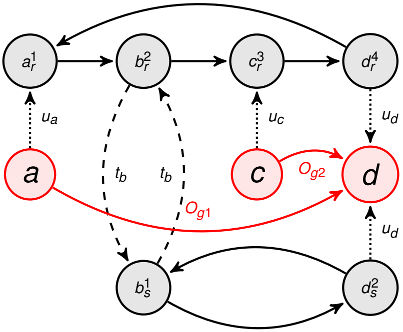
$$\sum_{r \in \mathbf{R}: v_r = v} Y^r \leq z^v \quad v \in \mathbf{V} \tag{14}$$

$$\tag{15}$$

$$x^p \in \mathbb{R}^+ \quad p \in P_g, g \in \mathbf{G} \tag{16}$$

$$O_g \in \mathbb{R}^+ \quad g \in \mathbf{G} \tag{17}$$

$$Y^r \in \mathbb{Z}^+ \quad r \in \mathbf{R} \tag{18}$$



Observation # 1

(Yet another) complex formulation

- Exponential number of route variables
- Exponential number of path variables

Mostly applicable for heuristic methods

Nested column generation?

Observation # 2

Easy to add transit time restrictions

- Path flow solved by column generation
- Transit time restrictions are in the subproblem
- Resource constrained shortest path problem

Observation # 3

Easy evaluation of a new solution

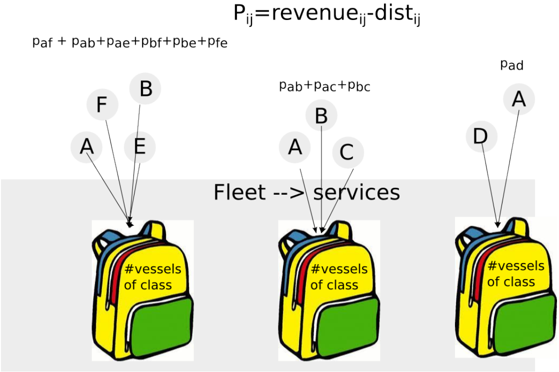
- Heuristic context
- Move $\Rightarrow$ changes a subset of edges
- Invalidated path variables removed
- Warm start with a nearly optimal basis

A composite matheuristic for the LSNDP-TT

1. Construction heuristic with multiple restarts
2. Improvement heuristic
3. Reinsertion heuristic
4. Perturbation heuristic

Construction heuristic

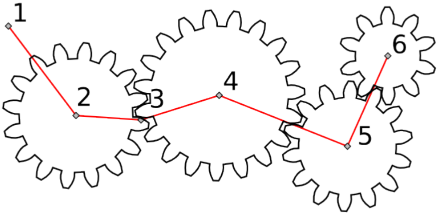
LSNDP interpreted as a multiple quadratic knapsack problem.



- Greedy algorithm
- Multiple restarts

Improvement heuristic

Fine tuning each service



- IP program as neighbourhood
- Decision variables: insert/ remove port calls, # vessels assigned

Insertion of port calls

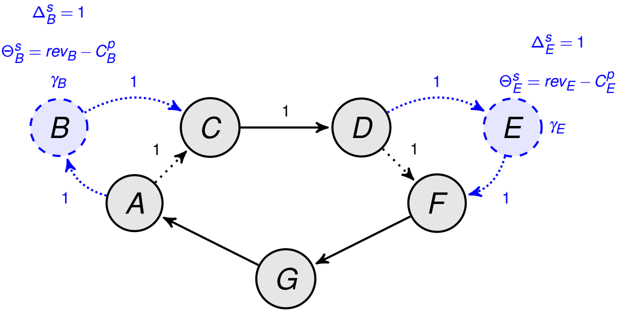


Figure: Blue nodes are evaluated for insertion - variables γ_i

- Estimation of distance increase (Δ_i^s)
- Estimation of profit (Θ_i^s)

Removal of port calls

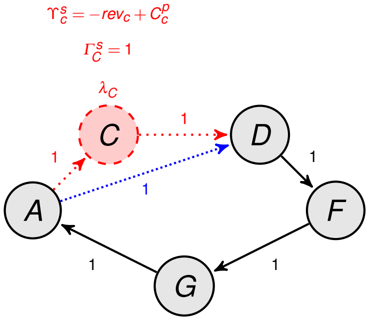


Figure: Red nodes are evaluated for removal - variables λ_i

- Estimation of distance decrease (Γ_i^s)
- Estimation of profit (Υ_i^s)

IP for improvement heuristic

$$\max \quad \sum_{i \in N^s} \Theta_i \gamma_i + \sum_{i \in P^s} \Upsilon_i \lambda_i - C_V^{e(s)} \omega_s \tag{19}$$

$$\text{subject to:} \quad \frac{D_s}{K_s} + \sum_{i \in P^s} B_{p(i)} + \sum_{i \in N^s} \left(\frac{\Delta_i^s}{K_s} + B_{p(i)}\right)\gamma_i - \sum_{i \in P^s} \left(\frac{\Gamma_i^s}{K_s} + B_{p(i)}\right)\lambda_i \le 24 \cdot 7 \cdot (n_s^e + \omega_s) \tag{20}$$

$$\omega_s \le M_{e(s)} \tag{21}$$

$$\sum_{i \in N^s} \gamma_i \le I_s \tag{22}$$

$$\sum_{i \in P^s} \lambda_i \le R_s \tag{23}$$

$$\sum_{j \in L_i} \lambda_j \le |L_i|(1 - \gamma_i) \qquad \forall i \in N^s \tag{24}$$

$$\sum_{j \in L_i} \lambda_j \le |L_i|(1 - \lambda_i) \qquad \forall i \in P^s \tag{25}$$

$$\lambda_i \in \{0, 1\} \qquad \forall i \in P^s \tag{26}$$

$$\gamma_i \in \{0, 1\} \qquad \forall i \in N^s \tag{27}$$

$$\omega_s \in \mathbb{Z} \tag{28}$$

IP for improvement heuristic

$$\begin{aligned}
&\mathit{max} && \text{est. \$ of insertion} + \text{est. \$ of removal} - \text{\$ of est. vessel adjustment} && (29)\\
&\text{subject to:} && \text{Current duration + additional time for insertion-saved time from removals} \leq && (30)\\
& && \text{num weeks of assigned vessels +vessel adjustment} && \\
& && \text{vessel adjustment} \leq \text{vessels available} && (31)\\
& && \text{Max insertions} \leq I_s && (32)\\
& && \text{Max removals} \leq R_s && (33)\\
& && \text{if insertion of i lock related port calls} \quad \forall i \in N^s && (34)\\
& && \text{if removal of i lock related port calls} \quad \forall i \in P^s && (35)\\
& && \lambda_i \in \{0,1\} \quad \forall i \in P^s && (36)\\
& && \gamma_i \in \{0,1\} \quad \forall i \in N^s && (37)\\
& && \omega_s \in \mathbb{Z} && (38)
\end{aligned}$$

Adjusting for transit time restrictions

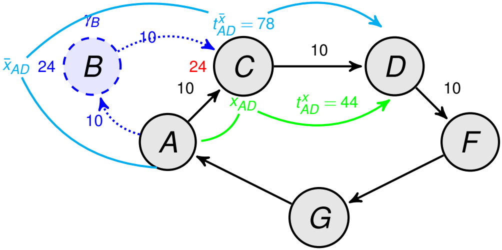


Figure: Insertions/removals affect transit time of current flow

- Commodity g_{AD} has a maximum transit time of 48 hours
- Insertion of γ_B will make path variable x_{AD} infeasible

Adjusting the IP to accound for transit time

$$\begin{aligned}
max \quad & \sum_{i \in N^s} \Theta_i \gamma_i + \sum_{i \in P^s} \Upsilon_i \lambda_i - C_V^{e(s)} \omega_s - \zeta_x \rho_x && && (39) \\
\text{subject to:} \quad & \sum_{i \in N^x} \left(\frac{\Delta_i^s}{K_s} + B_{p(i)}\right)\gamma_i - \sum_{i \in P^x} \left(\frac{\Gamma_i^s}{X_s} + B_{p(i)}\right)\lambda_i - UB\rho_x \leq s_x && \forall x \in X^s && (40) \\
& \rho_x \in \{0, 1\} && \forall x \in X^s && (41) \\
& && && (42)
\end{aligned}$$

- ζ_x: Estimated penalty for cargo lost due to transit time
- s_x: Slacktime of path variable x
- (40): Estimate transit time violations for path variable x of commodity g

Reinsertion heuristic

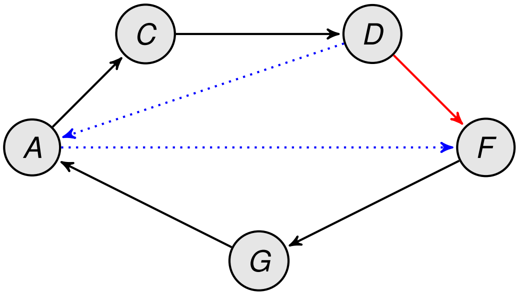


Figure: Reinsertion of node A to form a butterfly route

- Red edge is fully utilized by multiple commodities e.g. g_{AF}, g_{AG}
- Commodity g_{AF} is not transported in full
- Reinsertion (blue edges) allows transport of entire g_{AF} and decreases transit time of e.g. g_{DA}, g_{AF}, g_{AG}

Perturbation heuristic

Alter composition of services

- Search solutions with a different number of services of each vessel class
- Remove services with lowest utilization percentage
- Apply construction heuristic on excess fleet

Overview of the matheuristic for the LSNDP

Require: An instance (P, K, E, D) of the LSNDP

Apply construction heuristic to obtain an initial solution x.

Set the best known solution $x^* = x$.

Set the iteration counter $iter = 0$.

while no stopping criterion is met **do**

Search for an improved solution x' using the improvement heuristic.

if Successful **then**

Set $x \leftarrow x'$

Possibly update best known solution to current: $x^* \leftarrow x$

$iter \leftarrow iter + 1$

if $iter \mod 4 = 0$ **then**

Apply reinsertion heuristic to yield a new solution x' with promising butterfly routes.

if Successful **then**

Set $x \leftarrow x'$

Possibly update best known solution to current: $x^* \leftarrow x$

if $iter \mod 10 = 0$ **then**

Apply perturbation to obtain a solution x' with a different service composition.

Set $x \leftarrow x'$

Possibly update best known solution to current: $x^* \leftarrow x$

return (x^*)

Computational results - **LINER-LIB2012**

Instance	$\lvert P\rvert$	$\lvert G\rvert$	$\lvert E\rvert$	**v**
Baltic	12	22	2	5
WAF	19	38	2	42
Mediterranean	39	369	3	20
Pacific	45	722	4	102
WorldSmall	47	1764	6	263
AsiaEurope	111	4000	6	176

LINER-LIB2012 - |**P**|: Number of ports;|**G**|: Number of commodities; |**E**|: Number of vessel classes;**v**: Total number of vessels in base capacitated case

The very first results for LSNDP-TT

Instance		Z	Depl%	Trans%	Time	Max time
Baltic	Base	$-5.42 \cdot 10^6$	100.0	92.1	93.74	300
WAF	Base	$-1.01 \cdot 10^8$	95.2	94.1	210.9	900
Mediterranean	Base	$5.6 \cdot 10^7$	85	81.1	387.5	1800
Pacific	Base	$3.03 \cdot 10^8$	77	71.6	3601.1	3600
WorldSmall	Base	$-3.60 \cdot 10^8$	90.5	79.6	10413.4	10400
AsiaEurope	Base	$-3.85 \cdot 10^8$	91.5	80.7	14443.1	14400

Table: Best of 5 runs on an *Intel(R) Xeon(R) X5550* CPU at *2.67GHz* with *24 GB* RAM. Objective value (**Z**); percentage of fleet deployed (**Depl%**); percentage of total cargo volume transported (**Trans%**); execution time in CPU seconds (**Time**) and maximum execution time allowed (**max time**)

Promising results

4 of 6 instances profitable

The fleet deployment percentage is very low

Perturbation heuristic needs additional work

Conclusions

- Reference model formulated with a path flow formulation
- Transit time restrictions easily imposed
- Extend matheuristic to consider transit time restrictions
- Investigate the low fleet deployment
- Ideas are most welcome!

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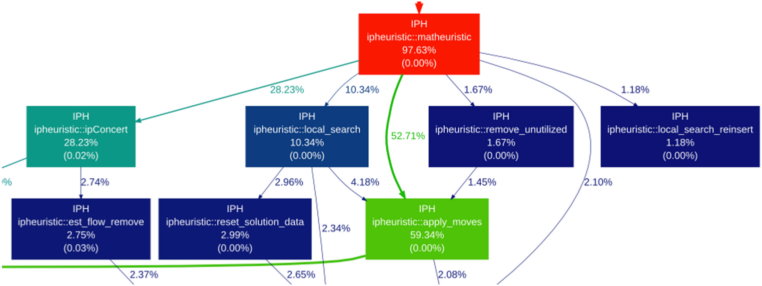
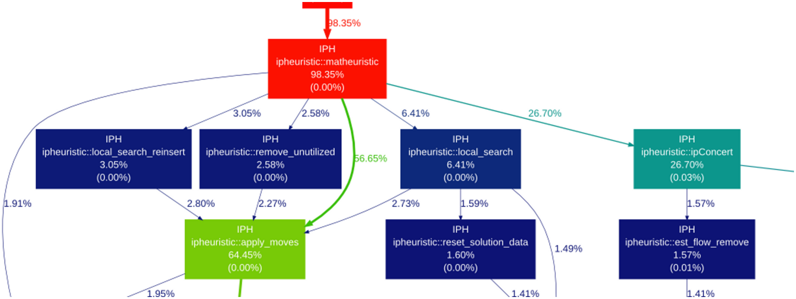
Profiling the code



- A larger part of the execution time is spent evaluating a solution
- Primarily due to increased complexity of the MCF
- Smaller percentage of time used in the perturbation heuristic

Profiling the code



- A larger part of the execution time is spent evaluating a solution
- Primarily due to increased complexity of the MCF
- Smaller percentage of time used in the perturbation heuristic

BUT!

Instance		Z	Depl%	Trans%	Time (CPU sec.)
Baltic	t=300	$-5.42 \cdot 10^6$	100.0	92.1	93.74
WAF	t=900	$-1.01 \cdot 10^8$	95.2	94.1	210.9
Mediterranean	t=1800	$5.6 \cdot 10^7$	85	81.1	387.5
Pacific	t=3600	$3.03 \cdot 10^8$	77.0	71.6	3601.1
	t=5400	$2.83 \cdot 10^8$	79.0	74.0	5402.0
WorldSmall	t=10400	$-3.60 \cdot 10^8$	90.5	79.6	10413.4
	t=18000	$-4.09 \cdot 10^8$	91.6	79.6	18013.9
AsiaEurope	t=14400	$-3.85 \cdot 10^8$	91.5	80.7	14443.1
	t=21600	$-3.85 \cdot 10^8$	91.5	80.7	21641

Increasing allowed execution timetime does not make a big difference!

The matheuristic on LSNDP and LSNDP-TT

Instance			Z	Depl%	Trans%	Time (CPU sec.)
Baltic	Base	LSNDP	$8.32 \cdot 10^5$	100.0	85.7	8.7
		LSNDP-TT	$-5.42 \cdot 10^6$	100.0	92.1	93.74
WAF	Base	LSNDP	$-1.38 \cdot 10^8$	95.2	94.9	58.8
		LSNDP-TT	$-1.01 \cdot 10^8$	95.2	94.1	210.9
Mediterranean	Base	LSNDP	$3.41 \cdot 10^7$	100	92.9	172.0
		LSNDP-TT	$5.6 \cdot 10^7$	85	81.1	387.5
Pacific	Base	LSNDP	$-6.19 \cdot 10^7$	96.0	94.8	3601.8
		LSNDP-TT	$3.03 \cdot 10^8$	77	71.6	3601.1
WorldSmall	Base	LSNDP	$-1.32 \cdot 10^9$	98.9	94.3	10447.2
		LSNDP-TT	$-3.60 \cdot 10^8$	90.5	79.6	10413.4
AsiaEurope	Base	LSNDP	$-6.75 \cdot 10^8$	98.9	92.4	14578.4
		LSNDP-TT	$-3.85 \cdot 10^8$	91.5	80.7	14443.1

Table: Results from matheuristic with LSNDP model and LSNDP-TT extension using LINER-LIB 2012. **Z**: Objective value (Note: Minimization); **Depl%** percentage of fleet deployed; **Trans%** percentage of total cargo volume transported and **Time (CPU sec.)** execution time in CPU seconds. Best of five runs with identical seeds

NOTE

The LSNDP solutions are not feasible for LSNDP-TT