Università degli Studi di Brescia

A matheuristic for the multi-vehicle inventory routing problem

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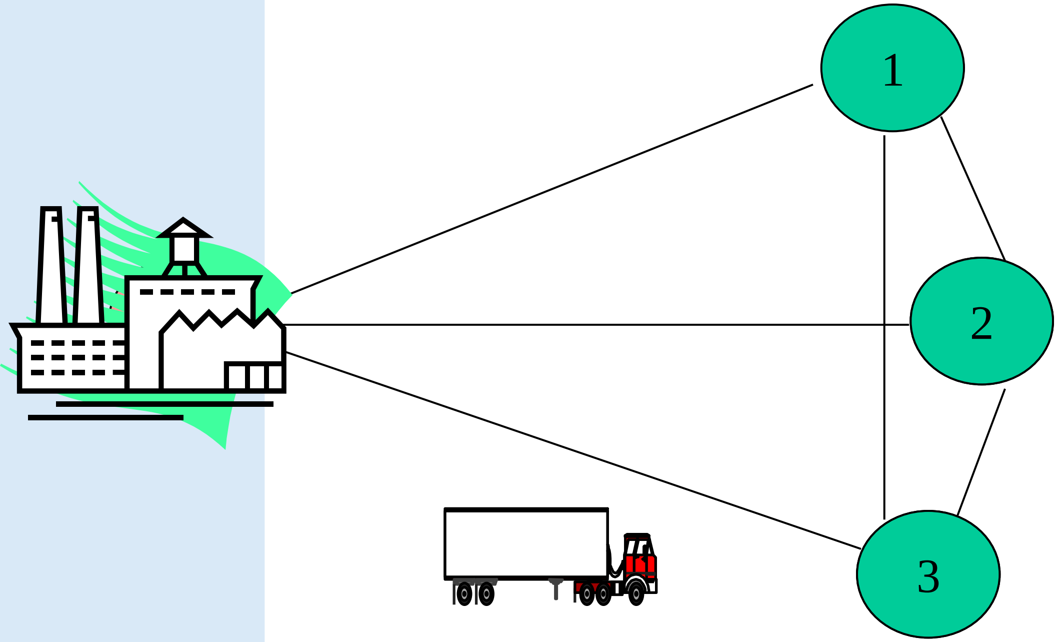
Inventory routing problems



Find

a distribution plan over a **period of time**

that minimizes

routing costs

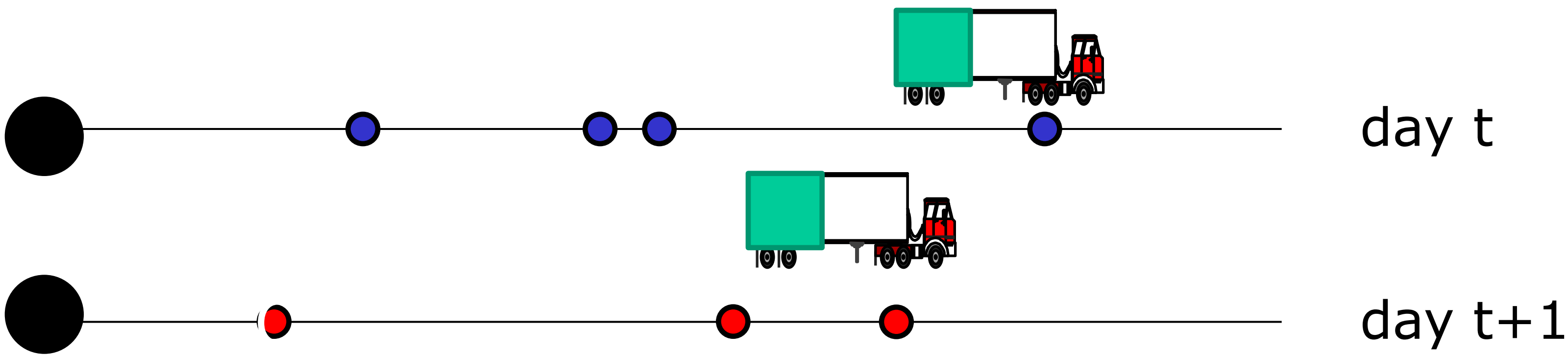
(and inventory holding costs)

My starting point at the end of '80

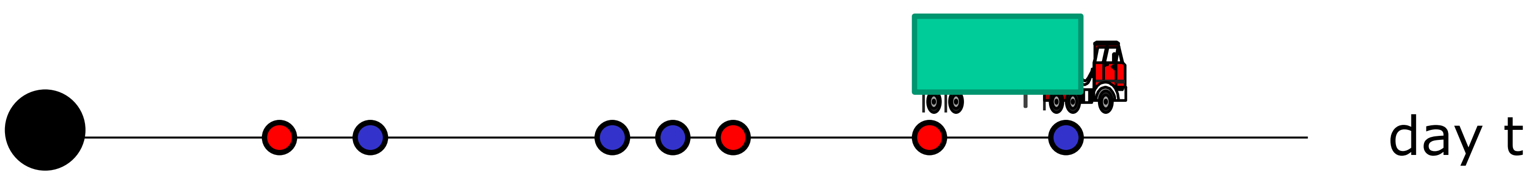
Inventory/transportation trade-off

Even without inventory and routing….

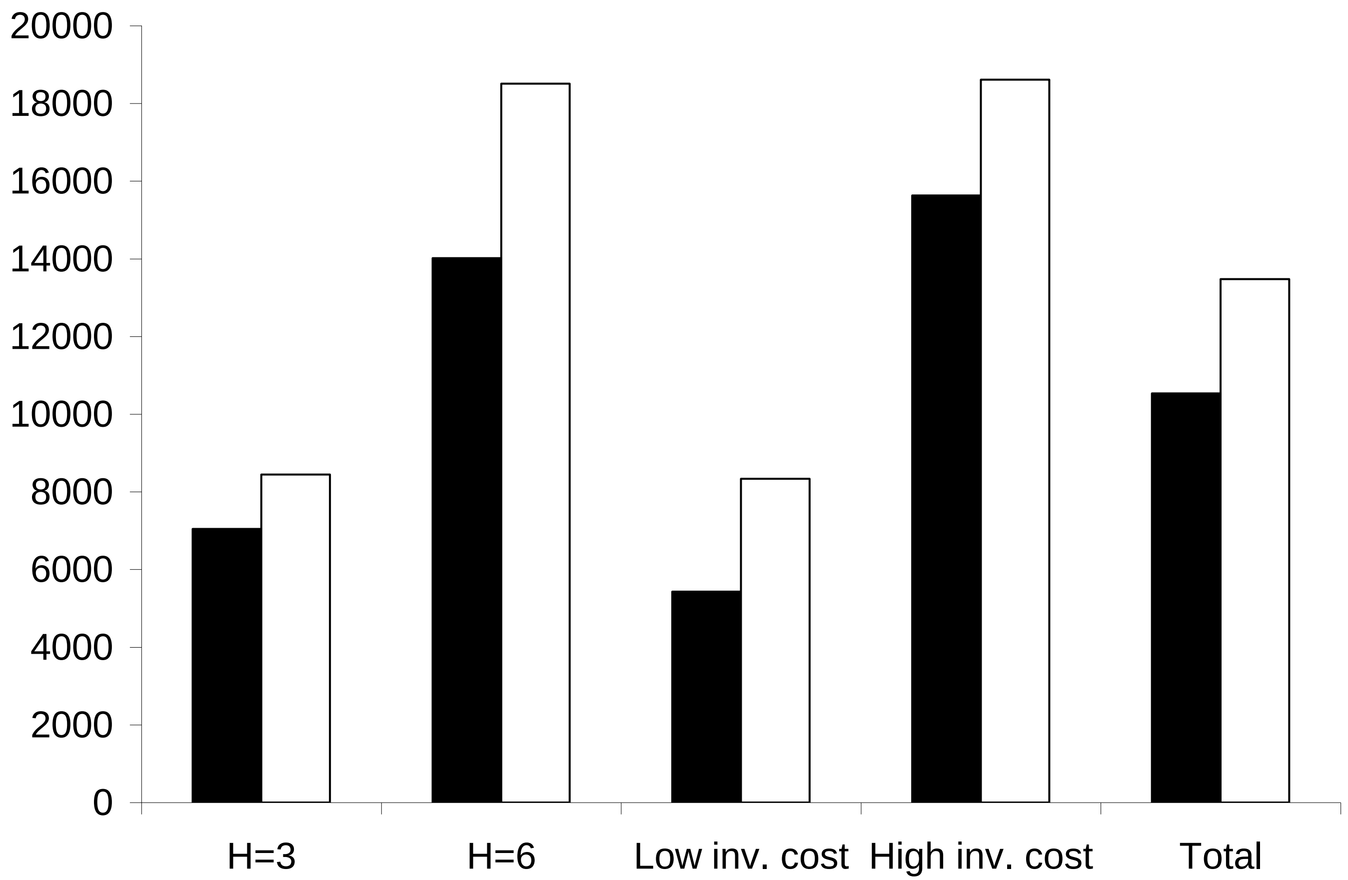
If time of service is fixed….



If time of service is flexible….



The value of IRPs



Without flexibility: average cost increase= 34%

Archetti, Speranza (2014)

The literature

- Surveys

Federgruen, Simchi-Levi (1995), ‘Handbooks in Operations Research and Management Science’, Ball et al (eds)

Campbell et al (1998), in ‘Fleet Management and Logistics’, Crainic, Laporte (eds)

Cordeau et al (2007), ‘Handbooks in Operations Research and Management Science: Transportation’, Barnhart, Laporte (eds)

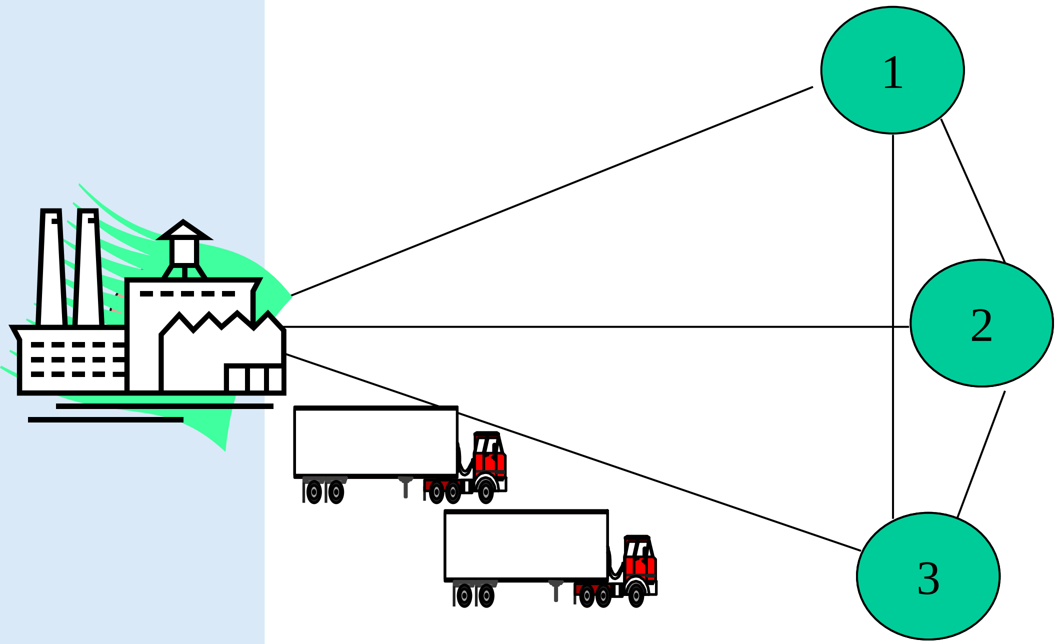
Bertazzi, Savelsbergh, Speranza (2008), ‘Vehicle routing: Latest advances and new challenges’, Golden, Raghavan, Wasil (eds)

Coelho, Cordeau, Laporte (2014), Transportation Science

- Tutorials

Bertazzi, Speranza (2012) and (2013), EURO Journal on Transportation and Logistics

Multi-vehicle IRP



- *n* customers
- *H* horizon
- *m* homogeneous vehicles with capacity *Q*
- Production rate at the supplier
- Daily demand at the customers
- Maximum inventory level at customers

Find the distribution plan:

Delivery schedule

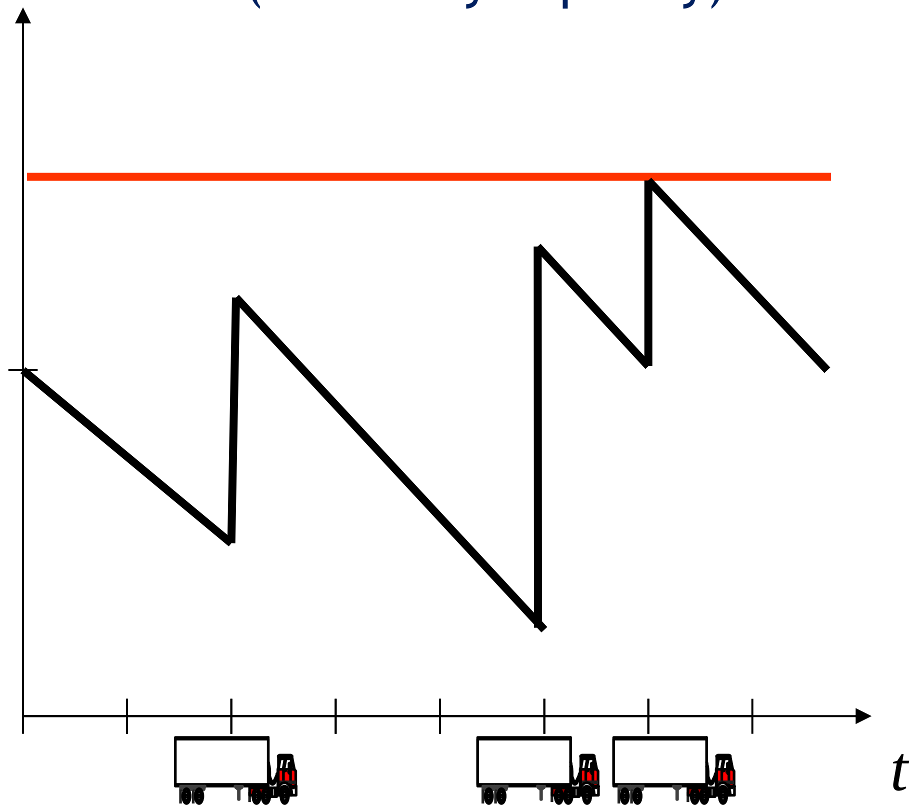
+

Routing

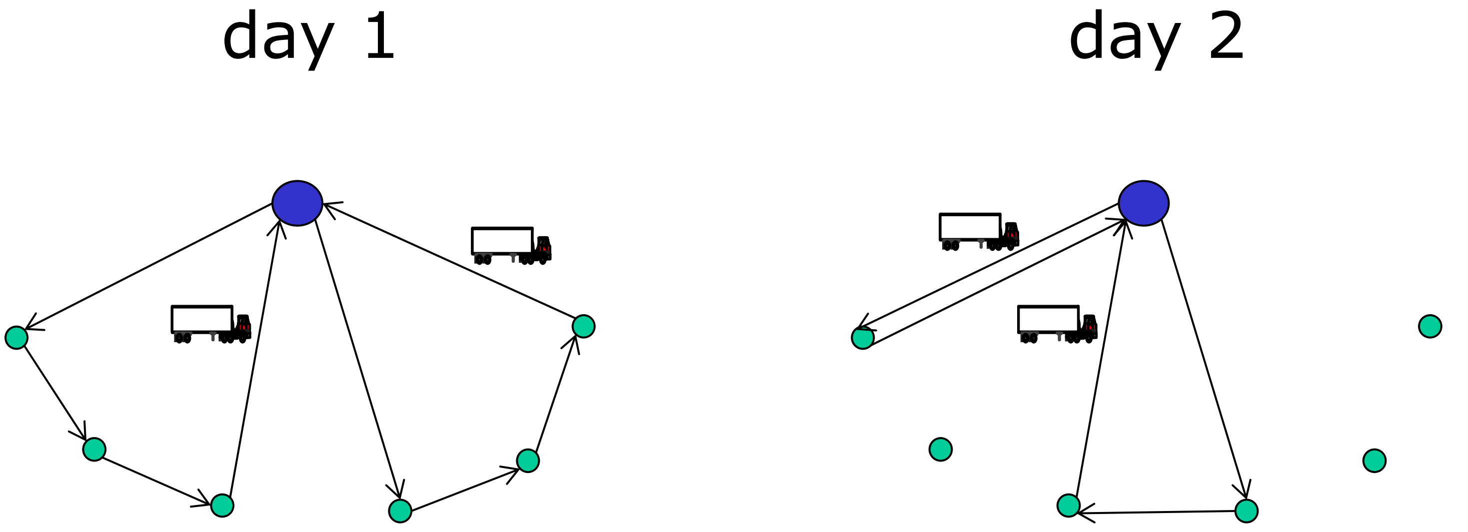
Minimizing the total cost

Replenishment policy: Maximum Level

Every time a customer is visited, the shipping quantity is such that **at most** the maximum level (inventory capacity) is reached



A solution



Mathematical formulation

Variables:

- Quantity to be delivered (horizon x customers x vehicles) - continuous
- Visit schedule (horizon x customers x vehicles) - binary
- Edge flow (horizon x customer^2 x vehicles) – binary (integer)

Objective function:

- Minimize Routing cost + Inventory holding cost at supplier and customers

Constraints:

- Inventory constraints
- Vehicle capacity constraints
- Routing constraints

Mathematical formulation for the MIRP

$$\min \sum_{t\in T} h_0 I_{0t} + \sum_{t\in T}\sum_{i\in N'} h_i I_{it} + \sum_{t\in T}\sum_{(i,j)\in E}\sum_{k\in K} c_{ij} y_{ij}^{kt}$$

$$I_{0t} = I_{0,t-1} + r_{0t} - \sum_{k\in K}\sum_{i\in N'} q_{it}^k \qquad t\in T$$

$$I_{it} = I_{i,t-1} - r_{it} + \sum_{k\in K} q_{it}^k \qquad t\in T, i\in N'$$

$$I_{it} \geq 0 \qquad t\in T, i\in N$$

$$\sum_{k\in K} q_{it}^k \leq U_i - I_{i,t-1} \qquad t\in T, i\in N'$$

$$q_{it}^k \leq U_i z_{it}^k \qquad t\in T, i\in N', k\in K$$

$$\sum_{k\in K} q_{it}^k \leq Q z_{0t}^k \qquad t\in T, k\in K$$

$$\sum_{k\in K} z_{it}^k \leq 1 \qquad t\in T, i\in N'$$

Mathematical formulation for the MIRP

$$\sum_{j:(i,j)\in E} y_{ij}^{kt} = 2z_{it}^{k} \qquad t\in T, i\in N, k\in K$$

$$\sum_{(i,j)\in E(S)} y_{ij}^{kt} \le \sum_{i\in S} z_{it}^{k} - z_{st}^{k} \qquad S\subseteq N', s\in S, t\in T, k\in K$$

$$z_{it}^{k}\in\{0,1\} \qquad t\in T, i\in N, k\in K$$

$$q_{it}^{k}\ge 0 \qquad t\in T, i\in N, k\in K$$

$$y_{ij}^{kt}\in\{0,1\} \qquad (i,j)\in E(N'), t\in T, k\in K$$

$$y_{0j}^{kt}\in\{0,1,2\} \qquad j\in N', t\in T, k\in K$$

and symmetry breaking constraints

Matheuristics

Matheuristics

Heuristics which include mathematical programming models, typically MILP models

Surveys

- Maniezzo, Stutzle, Voss (2010), Springer
- Ball (2011), SORMS

Matheuristics for routing

- Doerner, Schmidt (2010), Springer
- Archetti, Speranza (2013), submitted

A matheuristic for the Multi-vehicle IRP

Three phases

1. **Build an initial solution** (using a relaxation of the MILP formulation)
2. **Apply a tabu search**
3. **Improve the solution** (through a MILP)

Initial solution

Two different relaxations

1. (when n<30) Relax integrality constraints on edge flow variables
2. (when n≥30) Relax integrality constraints on edge flow variables and substitute them with aggregated variables measuring the flow of all vehicles on each edge

+

$$z_{it}^{k} - z_{jt}^{k} + y_{ij}^{t} \leq 1 \qquad (i,j) \in E, k \in K, t \in T$$

$$z_{jt}^{k} - z_{it}^{k} + y_{ij}^{t} \leq 1 \qquad (i,j) \in E, k \in K, t \in T$$

(if an edge is traversed, any vehicle must visit either none of the extremes or both)

Initial solution

- Any relaxation tells us when to visit customers, by which vehicle and the quantities to deliver (variables z_{it}^{k}, q_{it}^{k})
- The sequence of customers for each vehicle may be determined through any TSP algorithm (e.g., LK)

Initial solution

Greedy algorithm

(applied only in case no feasible solution is found for the relaxation within a time limit)

- Consider each customer sequentially from 1 to n
- Delivery times: as late as possible
- Delivery quantities: the minimum quantity necessary to avoid stockout
- Routes: Insert customers by means of cheapest insertion

The solution may be infeasible;
the tabu search will take care of it

Tabu search

- Search space: feasible solutions and infeasible solutions (violation of vehicle capacity or stock-out at the supplier)
- Solution value: total cost + two penalty terms
- Moves for each customer:
  - Removal of a day
  - Move of a day
  - Insertion of a day
  - Move into a different route
  - Swap with another customer
- After the moves: reduction of infeasibility
- Jump: when a certain number of iterations without improvement have elapsed, the current solution is destroyed and a new different solution is built

Improvement

A MILP model is solved to improve the best solution found by the tabu search

Two procedures

1. Procedure 1: solve a restricted version of the mathematical formulation of the MIRP where a subset of variables is set to 0 (when $n<30$)
2. Procedure 2: solve a route-based formulation of MIRP considering a subset of all feasible routes (when $n \geq 30$)

Improvement: Procedure 1

Choice of the subset of variables to be set to 0

1. Visit variables: if a customer *i* is rarely visited at time *t* by the solutions found by the tabu search, the corresponding variables are set to 0 for each vehicle *k*
2. Edge flow variables: if an edge (*i*,*j*) is rarely traversed by the solutions found by the tabu search, then variables related to the edge flow of (*i*,*j*) are set to 0, for each time *t* and each vehicle *k*

Improvement: Procedure 2

- Consider all the routes *R* generated by the tabu search (after removing dominated routes)
- Solve a MILP model which determines the best solution to the Multi-vehicle IRP using a subset of routes in *R*

Improvement: Procedure 2

$$\min \sum_{t \in T} h_0 I_{0t} + \sum_{t \in T} \sum_{i \in N'} h_i I_{it} + \sum_{t \in T} \sum_{r \in R} c_r x_{rt}$$

$$I_{0t} = I_{0,t-1} + r_{0t} - \sum_{r \in R} \sum_{i \in N'} q_{it}^r \qquad t \in T$$

$$I_{it} = I_{i,t-1} - r_{it} + \sum_{r \in R} q_{it}^r \qquad t \in T, i \in N'$$

$$I_{it} \geq 0 \qquad t \in T, i \in N$$

$$\sum_{r \in R} q_{it}^r \leq U_i - I_{i,t-1} \qquad t \in T, i \in N'$$

$$q_{it}^r \leq U_i z_{it}^r \qquad t \in T, i \in N', r \in R$$

$$\sum_{i \in N'} q_{it}^r \leq Q x_{rt} \qquad t \in T, r \in R$$

$$\sum_{r \in R} z_{it}^r \leq 1 \qquad t \in T, i \in N'$$

Improvement: Procedure 2

$$z_{it}^r \leq a_{it}^r x_{rt} \qquad t \in T, i \in N', r \in R$$

$$\sum_{t \in T} x_{rt} \leq 1 \qquad r \in R$$

$$\sum_{r \in R} x_{rt} \leq m \qquad t \in T$$

$$z_{it}^r \in \{0,1\} \qquad t \in T, i \in N', r \in R$$

$$q_{it}^r \geq 0 \qquad t \in T, i \in N', r \in R$$

$$x_{rt} \in \{0,1\} \qquad t \in T, r \in R$$

The computational tests

The computational time:

- 10 minutes for the solution of the relaxation
- 30 minutes for the tabu search
- 10 minutes for the improvement

- **Optimal solutions and upper bounds:**

  Coelho, Laporte (2013), C&OR

  (very powerful computer network, 84000 sec)

- **Heuristic solutions:**

  Adulyasak, Cordeau, Jans (2012), IJC

  (production routing)

Test instances – First set

160 benchmark IRP instances from Archetti et al. (2007) for the single vehicle case

- H = 3, n = 5,10, …., 50
- H = 6, n = 5,10, …., 30

modified for the Multi-vehicle IRP by setting

- |K|=2,…,5
- Q = vehicle capacity of the original instance divided by |K|

640 instances in total

- Optimal solutions and upper bounds from Coelho and Laporte (2013)
- Heuristic solutions from Adulyasak, Cordeau, Jans (2012) only for a subset of instances:
  - |K| = 2,3 for n=5,…,25
  - |K| = 3,4 for n=30,…,50 (only for horizon = 3)

First set: over 3 days

High inv. cost

200 inst.

148 opt.

	Aver. gap	Min gap	Max gap	#solutions
With respect to branch-and-cut optima	0.25	-	3.15	50 opt. (over 148)
With respect to branch-and-cut UB	-0.08	-13.12	5.81	31 better (over 200)
ALNS	-5.56	-13.92	0.01	89 better (over 100)

	Aver. gap	Min gap	Max gap	#solutions better
With respect to branch-and-cut optima	0.48	-	6.09	57 opt. (over 145)
With respect to branch-and-cut UB	-0.20	-15.30	6.58	34 better (over 200)
ALNS	-10.71	-24.50	0.00	88 better (over 100)

Low inv. cost

200 inst.

145 opt.

First set: over 6 days

High inv. cost

120 inst.

55 opt.

	Aver. gap	Min gap	Max gap	#solutions
With respect to branch-and-cut optima	0.31	-	2.82	20 opt. (over 55)
With respect to branch-and-cut UB	0.71	-8.81	5.64	14 better (over 120)
ALNS	-2.11	-6.03	1.46	43 better (over 50)

	Aver. gap	Min gap	Max gap	#solutions
With respect to branch-and-cut optima	0.56	-	3.39	18 opt. (over 55)
With respect to branch-and-cut UB	-0.33	-75.73	9.02	14 better (over 120)
ALNS	-3.77	-9.14	1.64	46 better (over 50)

Low inv. cost

120 inst.

55 opt.

Test instances – Large instances

240 instances generated with parameters as in Archetti et al. (2007) for the single vehicle case

- H = 6, n = 50, 100, 200
- High and low inventory cost
- |K|=2,…,5

Upper bounds from Coelho and Laporte (2013)
- no optimal solution found

Large instances: over 6 days, low inv. cost

	Aver. gap	Min gap	Max gap	#solutions better	#solutions =
n=50	-8.98	-34.96	1.68	31(40)	1(40)
n=100	-33.33	-52.08	-0.63	40(40)	0(40)
n=200	-49.42	-62.66	-18.88	40(40)	0(40)

	Aver. gap	Min gap	Max gap	#solutions better	#solutions =
m=2	-24.18	-51.52	1.68	23(30)	0(30)
m=3	-22.50	-47.72	0.82	28(30)	1(30)
m=4	-35.99	-58.50	-3.32	30(30)	0(30)
m=5	-39.63	-62.66	-1.31	30(30)	0(30)

Large instances: over 6 days, high inv. cost

	Aver. gap	Min gap	Max gap	#solutions better	#solutions =
n=50	-3.75	-22.13	1.28	29(40)	4(40)
n=100	-18.02	-26.69	-6.93	40(40)	0(40)
n=200	-23.14	-30.97	-13.89	40(40)	0(40)

	Aver. gap	Min gap	Max gap	#solutions better	#solutions =
m=2	-9.56	-22.40	1.28	23(30)	0(30)
m=3	-13.26	-24.66	0.00	27(30)	3(30)
m=4	-18.37	-30.58	0.00	29(30)	1(30)
m=5	-18.68	-30.97	-0.44	30(30)	0(30)

Conclusions

- First heuristic designed for the Multi-vehicle IRP
- Standard VRP schemes and operators are not sufficient to build effective solution methods
- MILP models help capturing the nature of the problem