

# A Routing and Reservation System for Battery Swaps for Electric Vehicles

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# Overview

Electric vehicles &  
Charging Infrastructure

Shortest Walk from O to D

Routing and Reservation System

# Electric vehicles

- Growing in popularity: Tesla sold over 22,450 electric cars in 2013
- Have a limited range, which can cause drivers to have anxiety
  - Typically charged at home or at the office for long periods of time
  - Charging at origin and destination insufficient for long range trips



Source: Tesla

# Charging an electric vehicle mid trip

## Fast charging stations

- A station where a car charges its battery quickly to a partially full state
- Still require a half an hour to charge
- Placed on the US Eastern and Western seaboards by Tesla, and can be used for free



## Battery exchange stations

- A station where a car swaps an empty battery with a fresh one
- Pioneered by Better Place, declared bankruptcy in May :(
- Expensive since many extra batteries are required to be at the stations
- Tesla has produced a vehicle that can battery swap in 90 seconds



Source: Better Place, Tesla

# Alternative-fuel vehicles

- Several different types of alternative fuels
  - Compressed Natural Gas (CNG)
  - Hydrogen fuel cells
- Specialized fuel requires specialized refueling stations, thus vehicles have similar problems as electric ones
- Toyota is rolling out hydrogen powered cars in California in 2015, CNG vehicles already available



Source: Toyota, Bizjournals.com

# Objective

- Optimization problems for design and operation of such vehicles are related to OR-type literature. E.g.,
  - **Routing vehicles from origin to destination (OD)**
  - Scheduling a fleet of vehicles to service customers
  - **Given OD demand, determining how the demand should be distributed along roads or constrained resources**
- Major Issue: Electric and alternative-fuel vehicles have a limited distance before they need to stop and refuel, which can only be done at a small number of locations
- *How can we solve these optimization problems for electric and alternative-fuel vehicles with fixed refueling locations?*

# The electric vehicle shortest walk problem

# Electric vehicle shortest walk problem

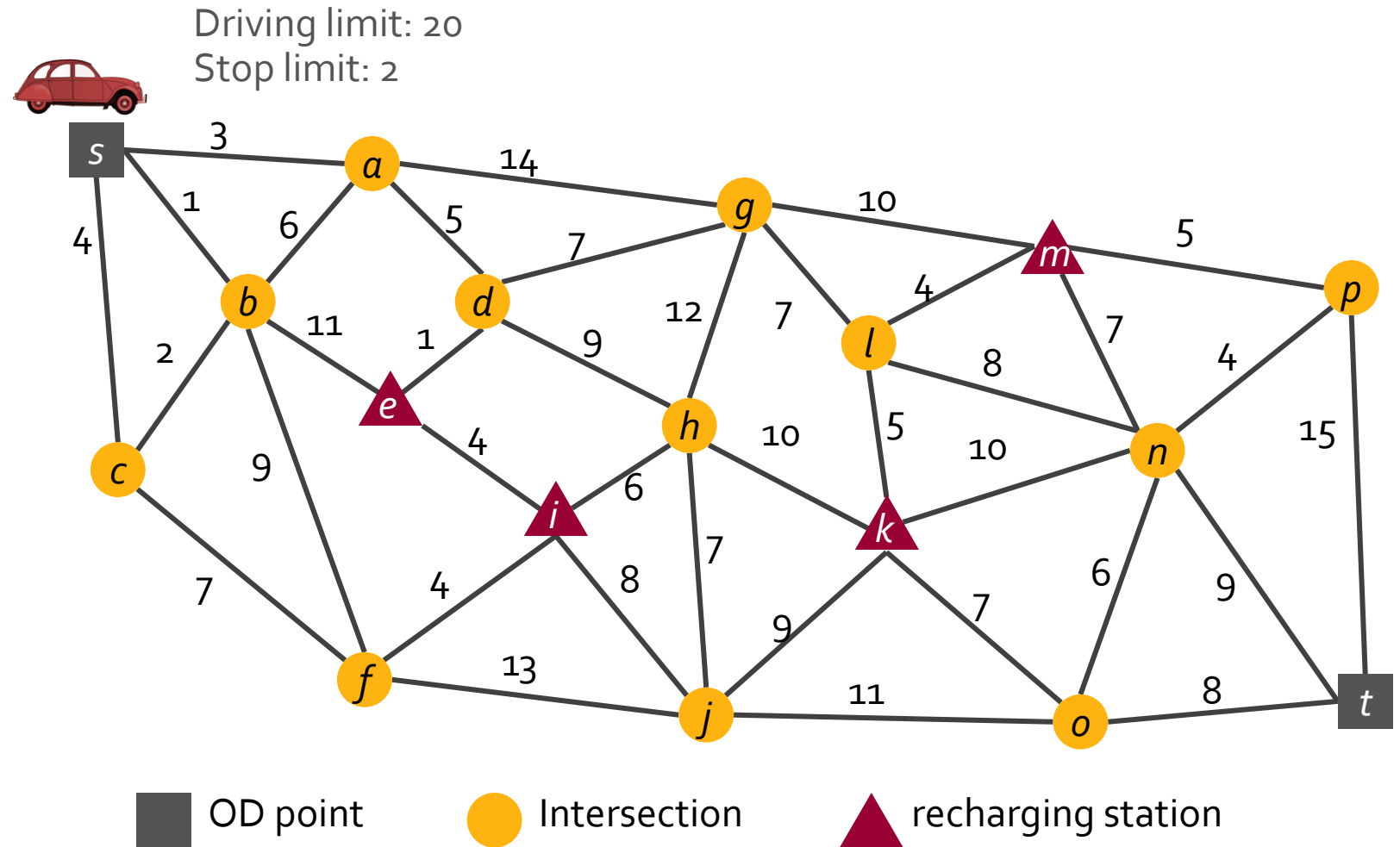
- Suppose we wanted to find the route an electric vehicle should take from an origin to a destination
- The route must include where to stop to recharge the battery
- Can't assume the shortest unconstrained path will have sufficient stops on the way
- Not necessarily a "path" since may have to traverse edges multiple times
- We may want to limit the number of stops to a certain number because they are frustrating
- *How do we find this shortest walk? Can it be done in polynomial time?*



# Brief Lit Review

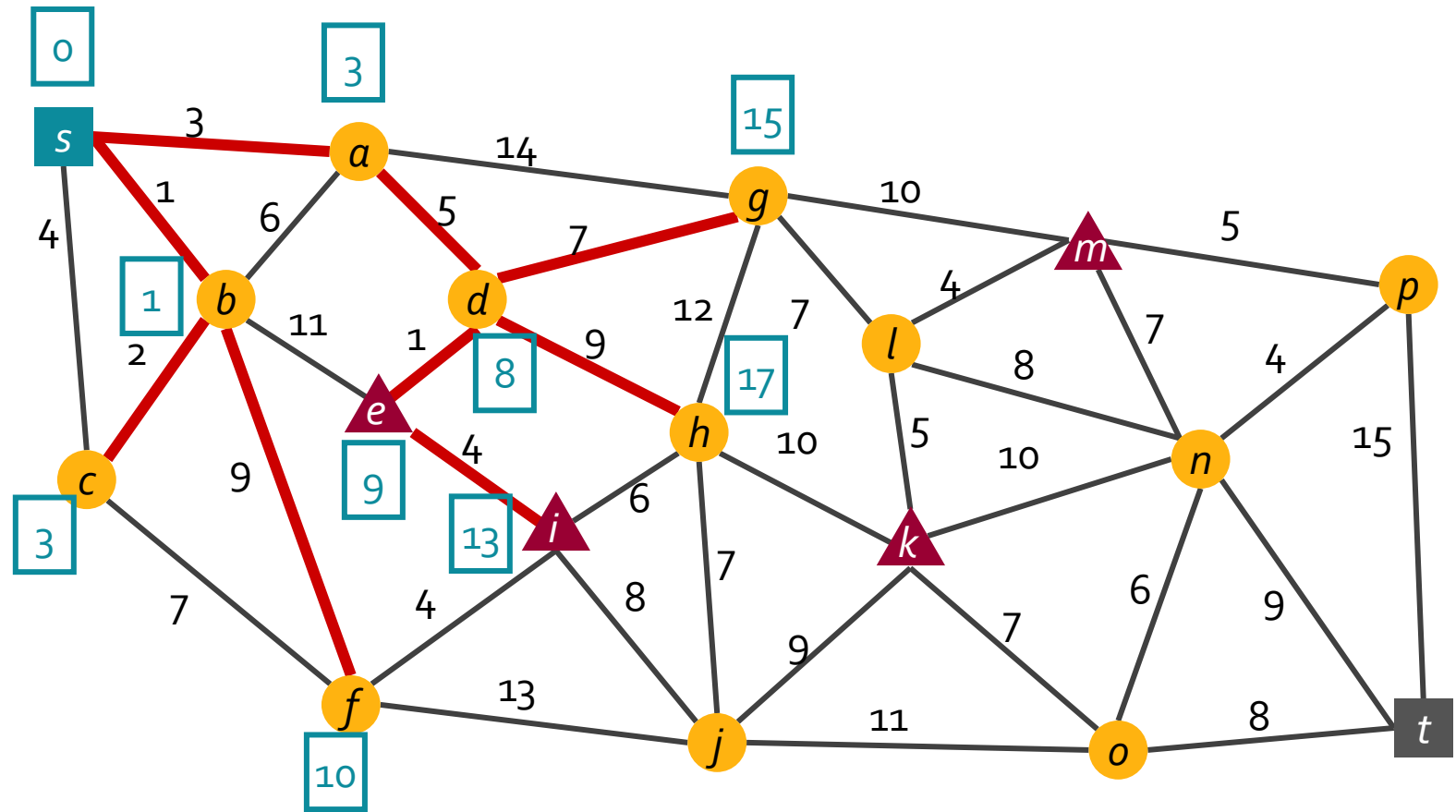
- Ichimori first analyzed this problem in 1981, didn't account for limiting the amount of times the vehicles stops
- We assume distance traveled and time are proportional, other people (Smith et al. 2012, Laporte & Pascoal 2011) analyzed the case where they are not
- Most modeling of where to locate charging stations (e.g., Kuby et al. 2005) assume no detouring

# Example Problem



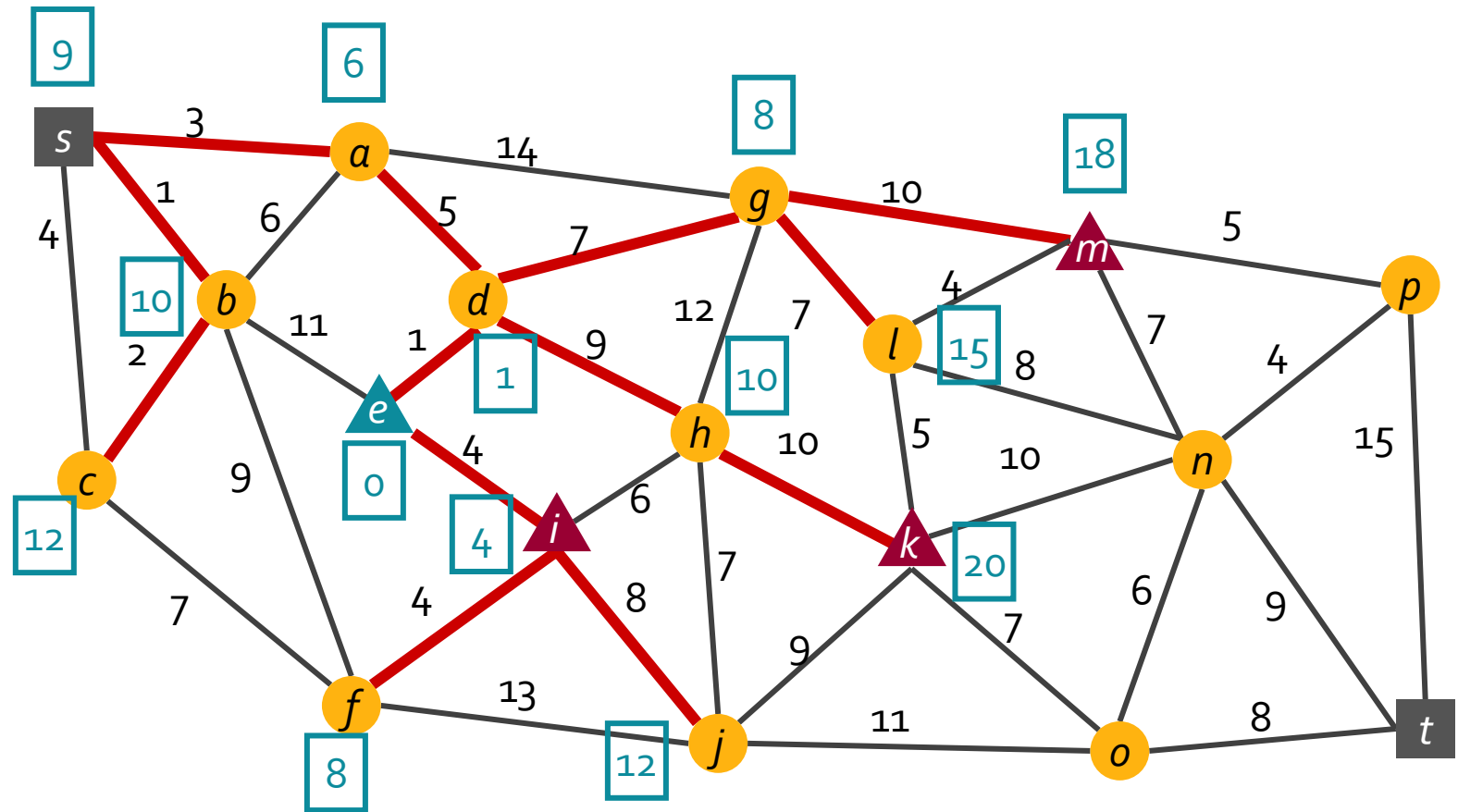
Objective is to get from  $s$  to  $t$  while stopping at most 2 times to charge the battery

# Spanning tree from *s*



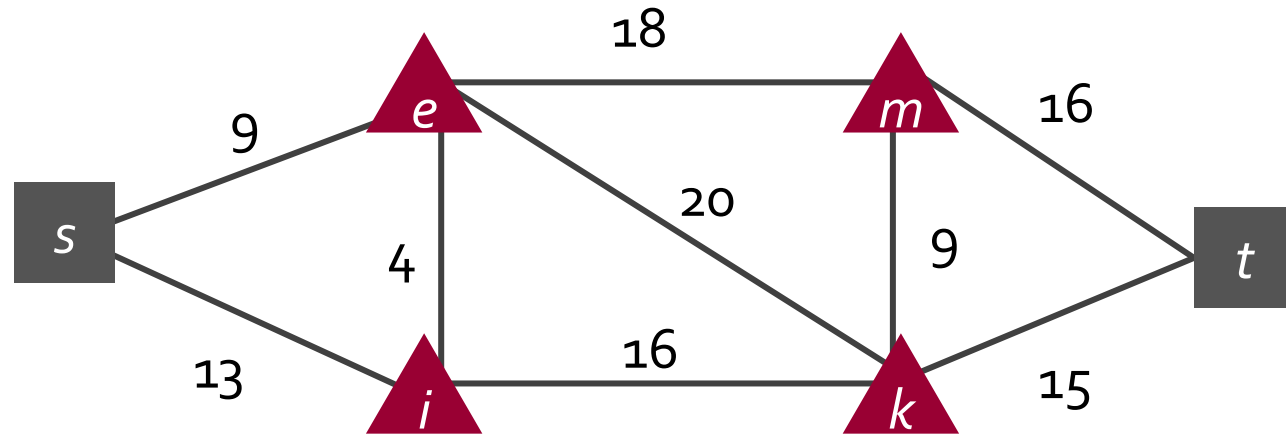
We can pre-calculate which charging stations are reachable from the start point

# Spanning tree from $e$



We can also calculate which charging stations are reachable from each other, and which can reach the terminal vertex

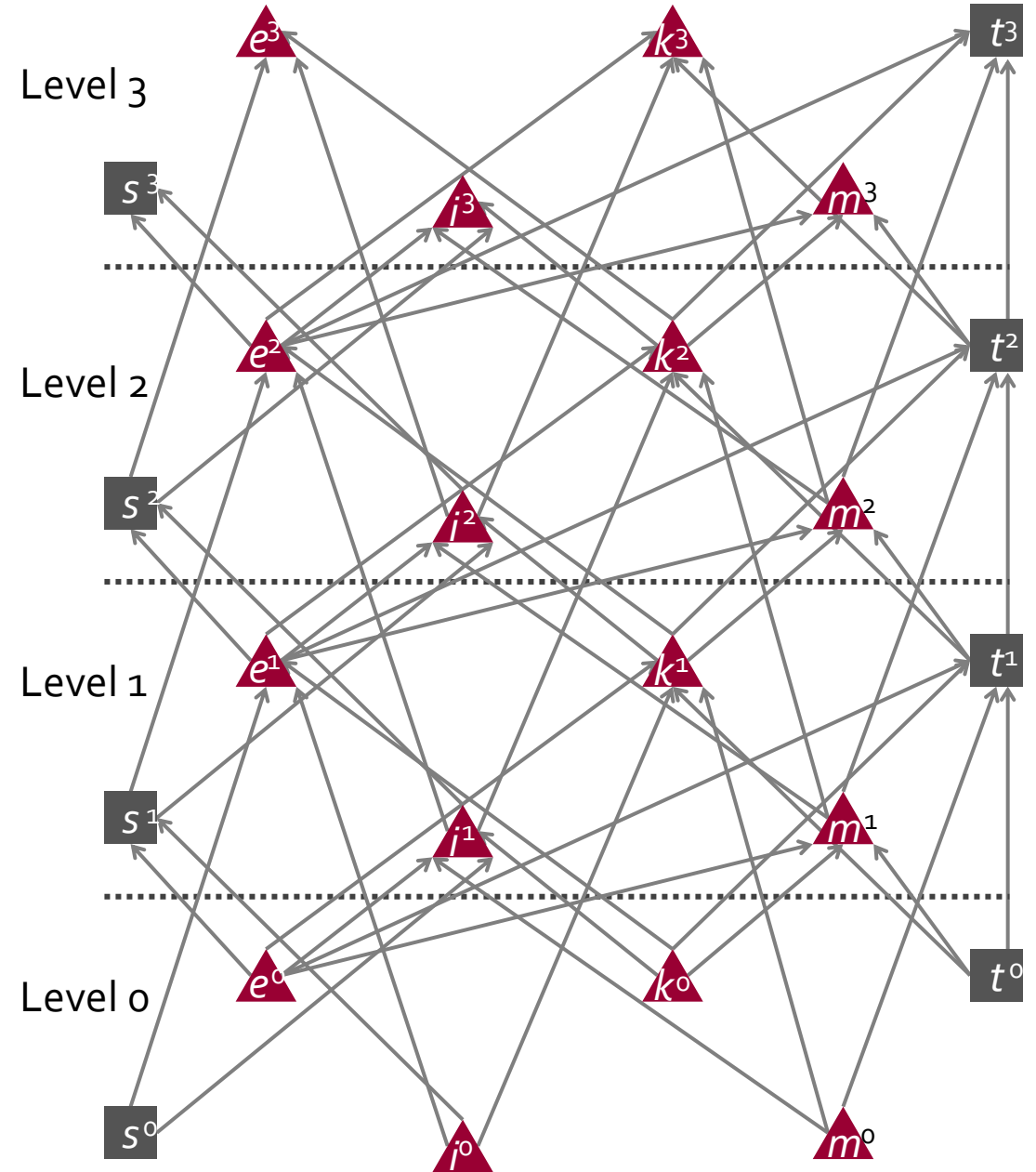
# Meta- network



- With all of those shortest paths, we can make a new meta-network
- The nodes in the meta-network have an edge if the vertices can be reached in a single charge in the original graph
- The shortest path in this graph corresponds to the shortest walk in the original graph without a stop limit

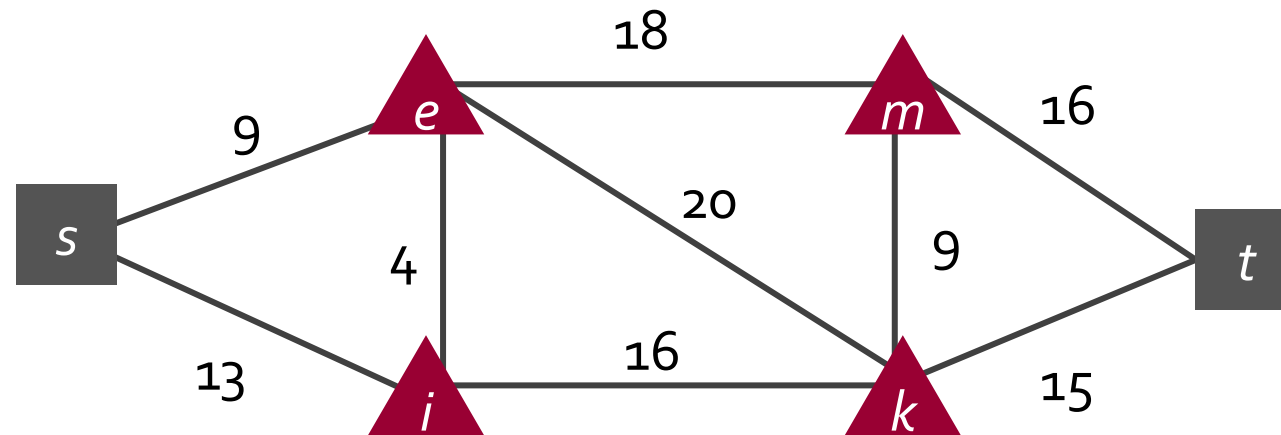
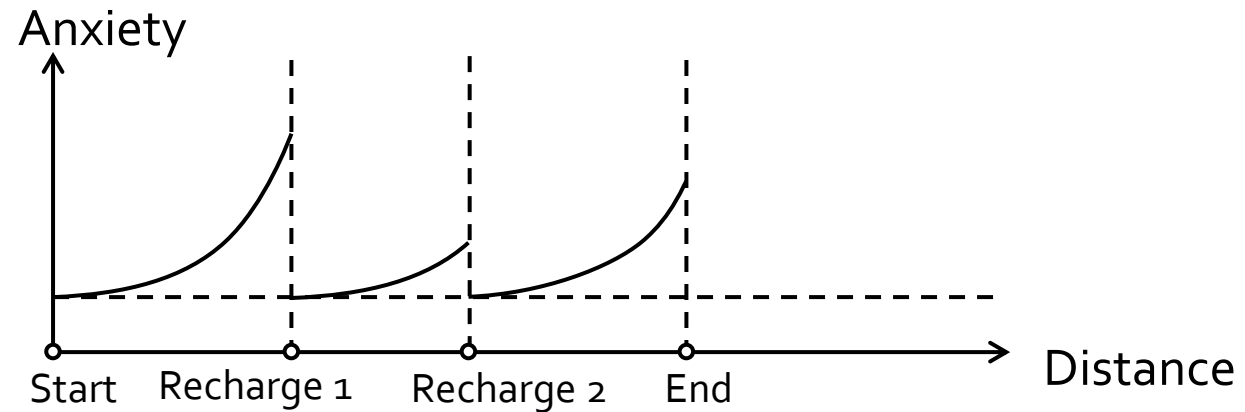
# Stop limited meta-network

- If there is a stop limit of  $p$ , then a graph with  $p + 2$  copies of the meta-network vertices should be generated
- An edge between  $(x_i^l, x_j^{l+1})$  exists if there is an edge  $(x_i, x_j)$  in meta-network, edges have the same cost
- $(t^l, t^{l+1})$  edges exist with 0 cost
- Polynomial time to get the shortest path



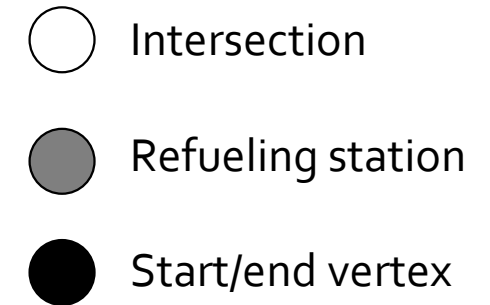
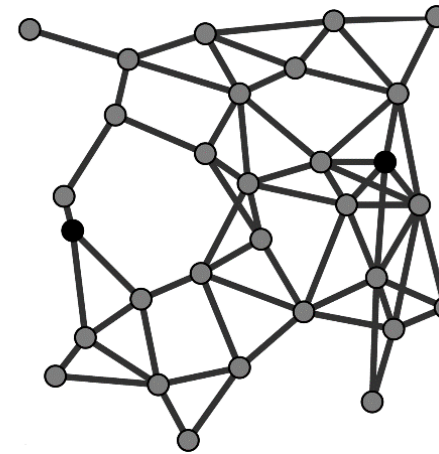
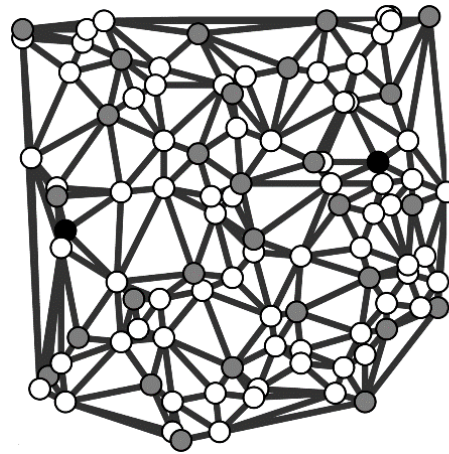
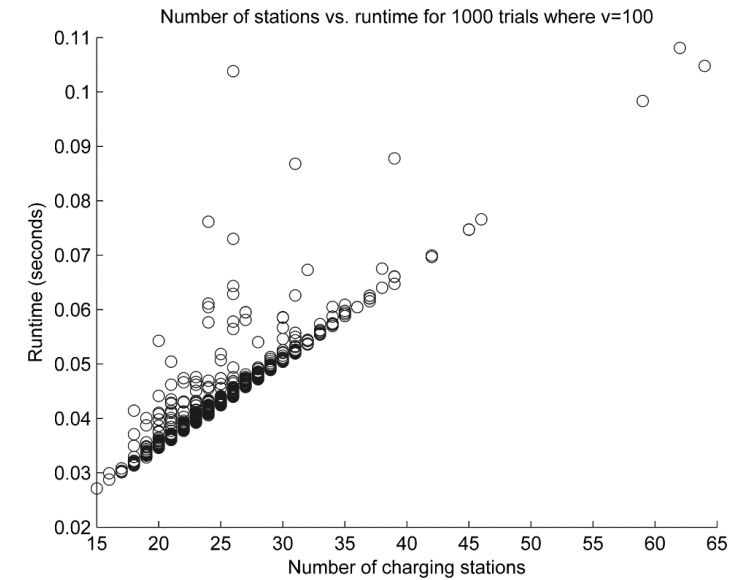
# Shortest anxiety walk

- A minimum anxiety walk minimizes the maximal path length between charging stations
- This generates the same meta-network (and multi-level meta-network), only now a modified Dijkstra's Algorithm needed to find best path



# Results

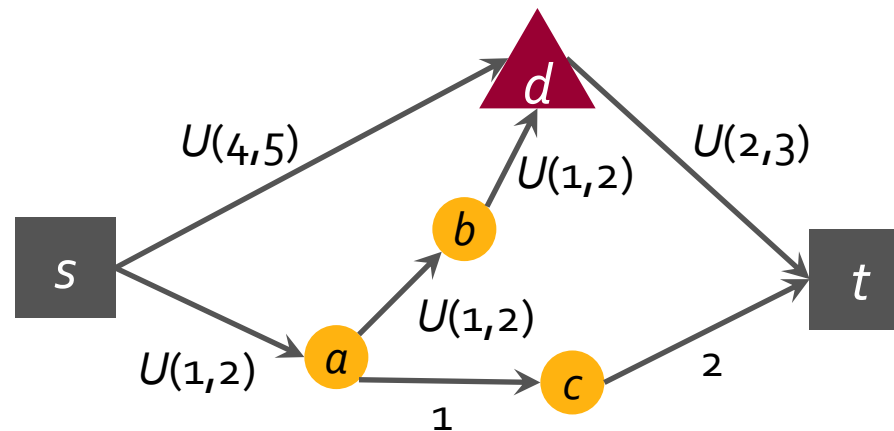
- Tested on randomly generated data
- Runtime grows polynomially with the number of stations (as expected through complexity analysis)






# Extensions

- What if the arc lengths are stochastic? Each edge has a known distribution and random outcome is selected each time it is traversed
- Now the walk may to be altered during the traversal depending on the realization
- Driver's appetite for risk needs to be incorporated in the model as well
- Can be modeled as a Markov decision process where the set of actions is limited to those that are sufficiently risk adverse





# Online routing and battery reservations for electric vehicles in a network with battery exchanges

# Battery exchange stations



Source: Better Place

# The steps in a routing and reservation system



Car turns on



Destination is inputted



The system plans a route



When driver accepts, batteries are reserved at stations

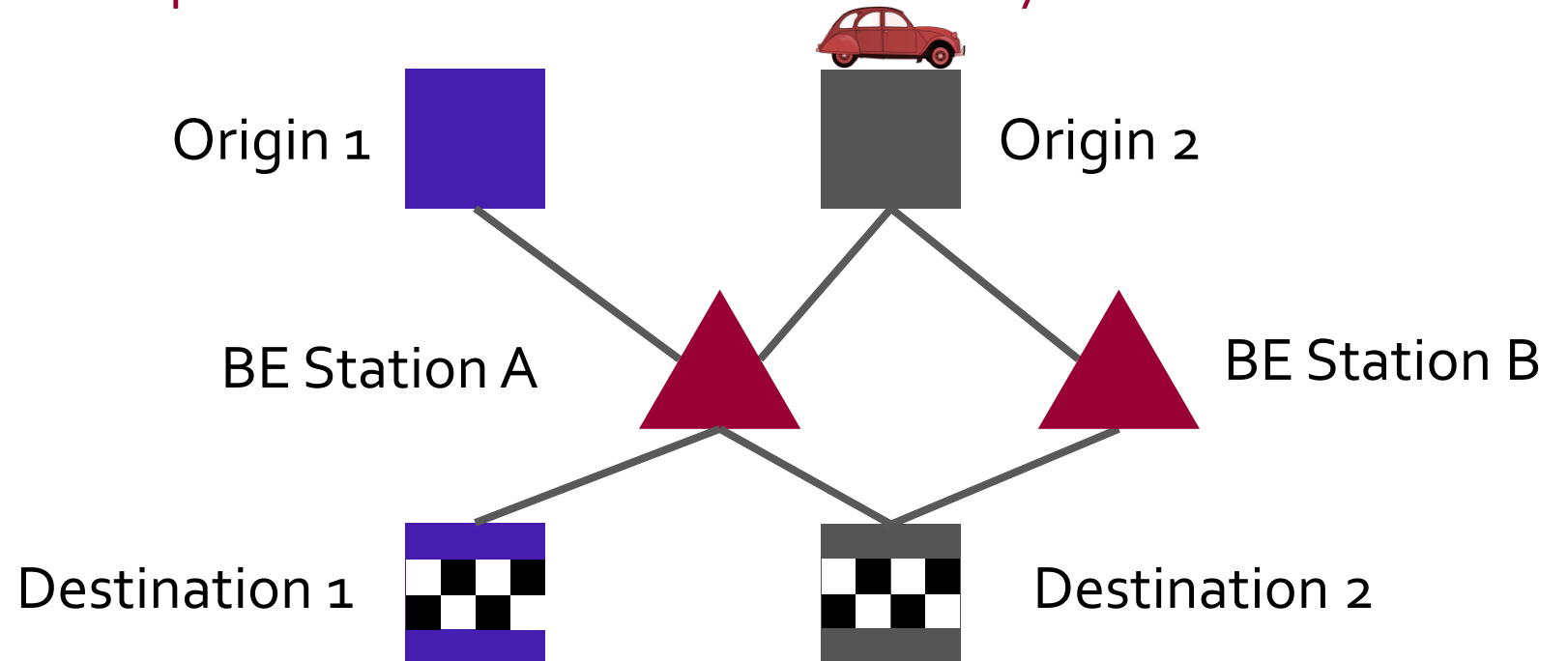


***DRIVE!***

# Problem statement

Once the battery exchange system is in place...

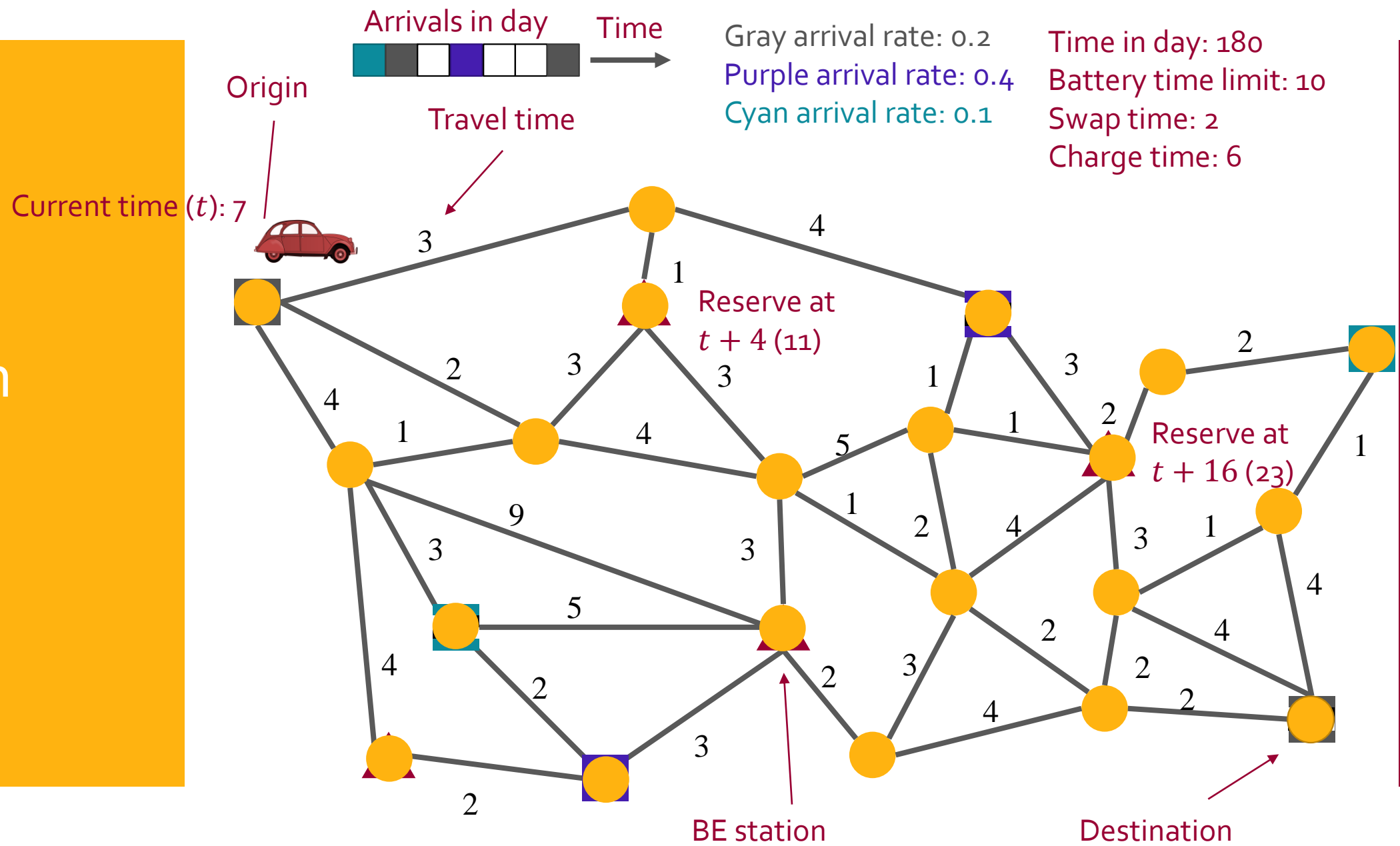
- If driver wants to make a trip given a current set of available batteries at stations, which route should they take?
- How do you route vehicles to minimize overall travel times?
- This depends on future arrivals into the system



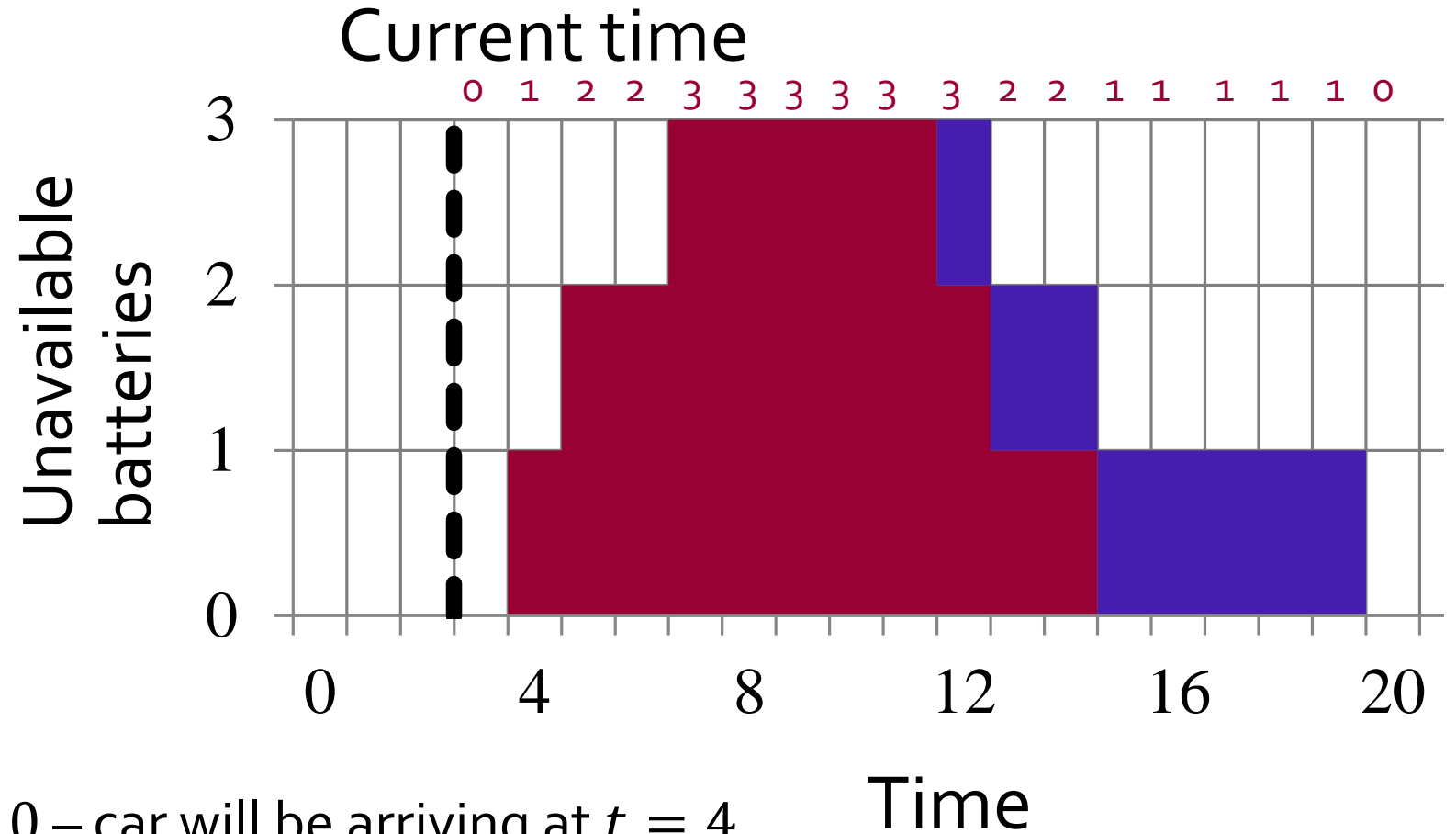
## Lit Review

- de Weert et al. (2013) routed multiple electric vehicles based on future demand, but didn't optimize globally
- Mak (2012) determined optimal routes for stochastic EV demand, but there was no online component
- Worley & Klabjan (2011) modeled when to recharge a station given stochastic demand, but had no network routing component

# Problem setup



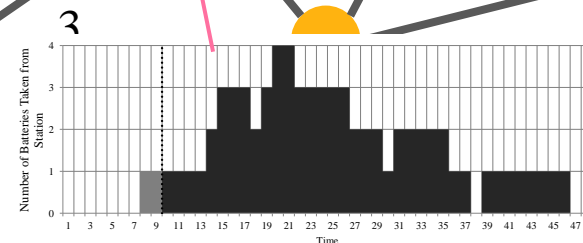
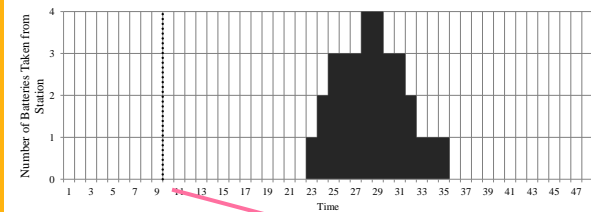
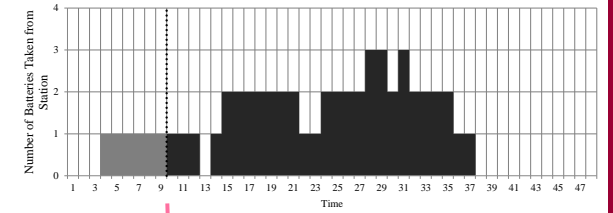
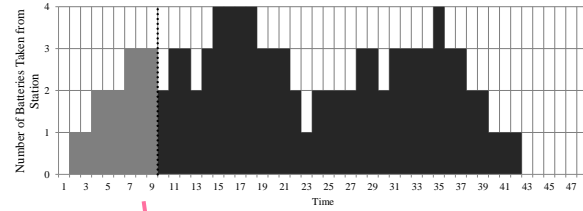
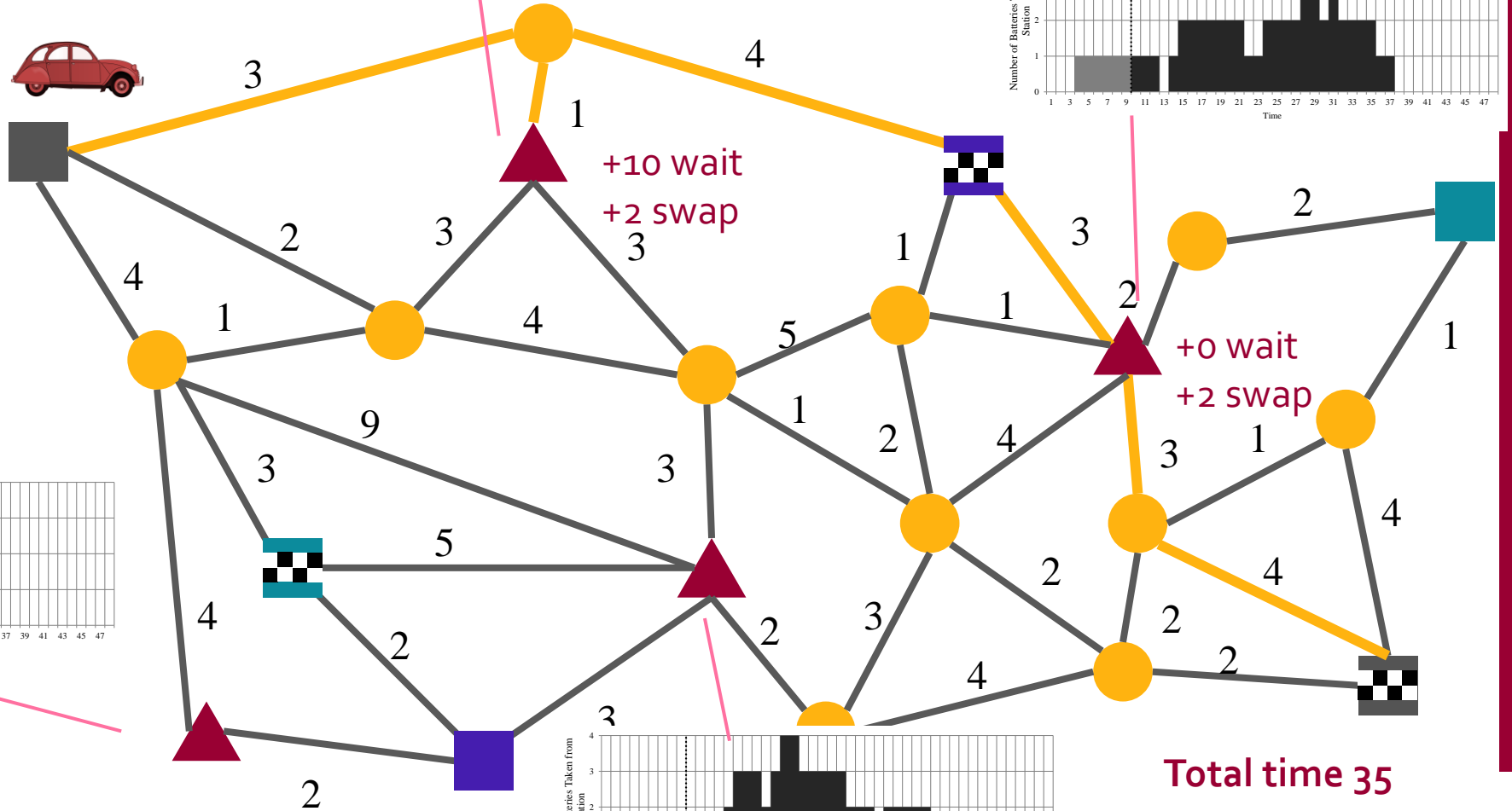
# Station information



- $t = 0$  – car will be arriving at  $t = 4$
- $t = 1$  – car will be arriving at  $t = 7$
- $t = 2$  – car will be arriving at  $t = 5$
- $t = 3$  – car will be arriving at  $t = 6$



# Network EV routing



Total time 35

# Objective

- A vehicle arriving at time  $t$  spends
  - $\psi_t^{drive}$  time units driving
  - $\psi_{t,i}^{swap}$  time units swapping batteries at station  $i$
  - $\psi_{t,i}^{wait}$  time units waiting at station  $i$
- And  $\psi_t^{optimal}$  is the optimal time to travel between the OD pairs that the vehicle at time  $t$

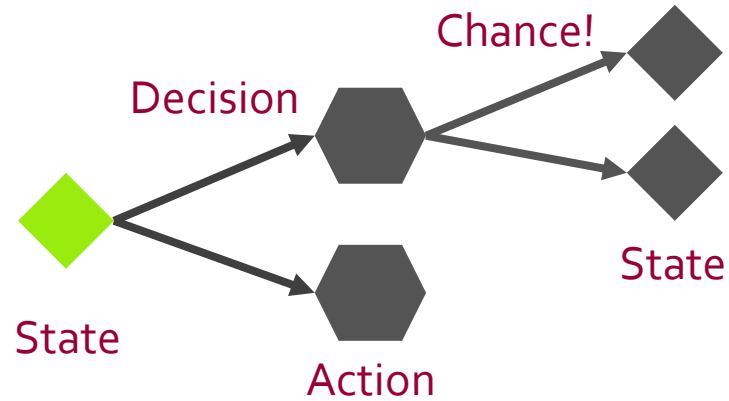
- The total delay for the vehicle arriving at time  $t$  is

$$\psi_t = \psi_t^{drive} + \rho_1 \sum_{i=1}^{\beta} \psi_{t,i}^{swap} + \rho_2 \sum_{i=1}^{\beta} \psi_{t,i}^{wait} - \psi_t^{optimal}$$

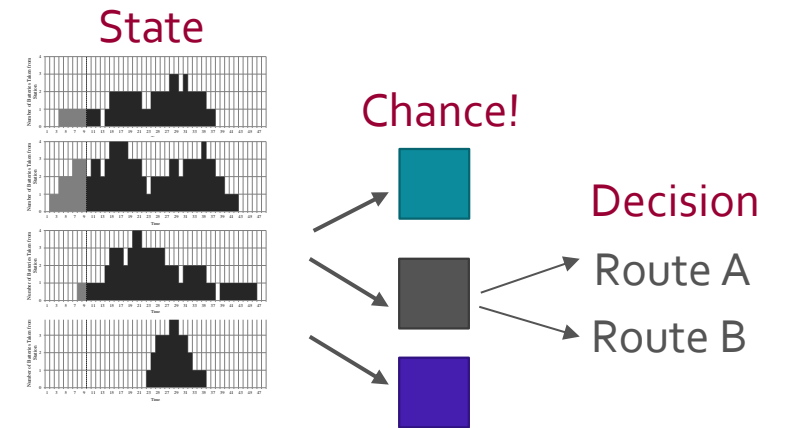
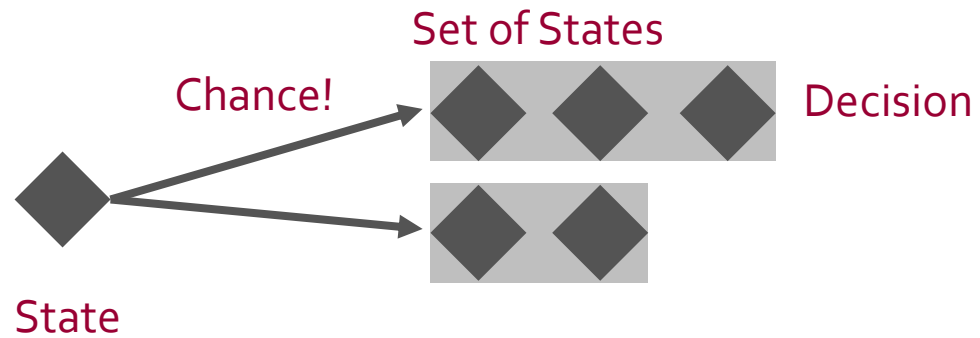
- To find a routing policy that minimizes the total delay  
 $\mathbb{E}[\psi_0 + \mathbb{E}[\psi_1 + \mathbb{E}[\psi_2 + \dots]]]$

# A Markov chance decision system

## Markov Decision Problem



## Markov Chance Decision Problem



# Approximate Dynamic Programming

## Approximate dynamic programming

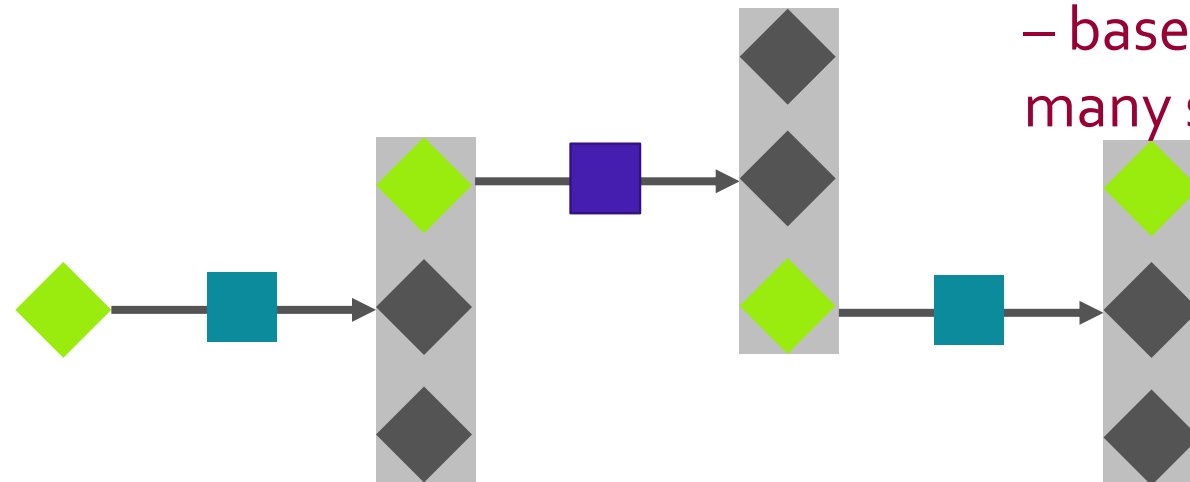
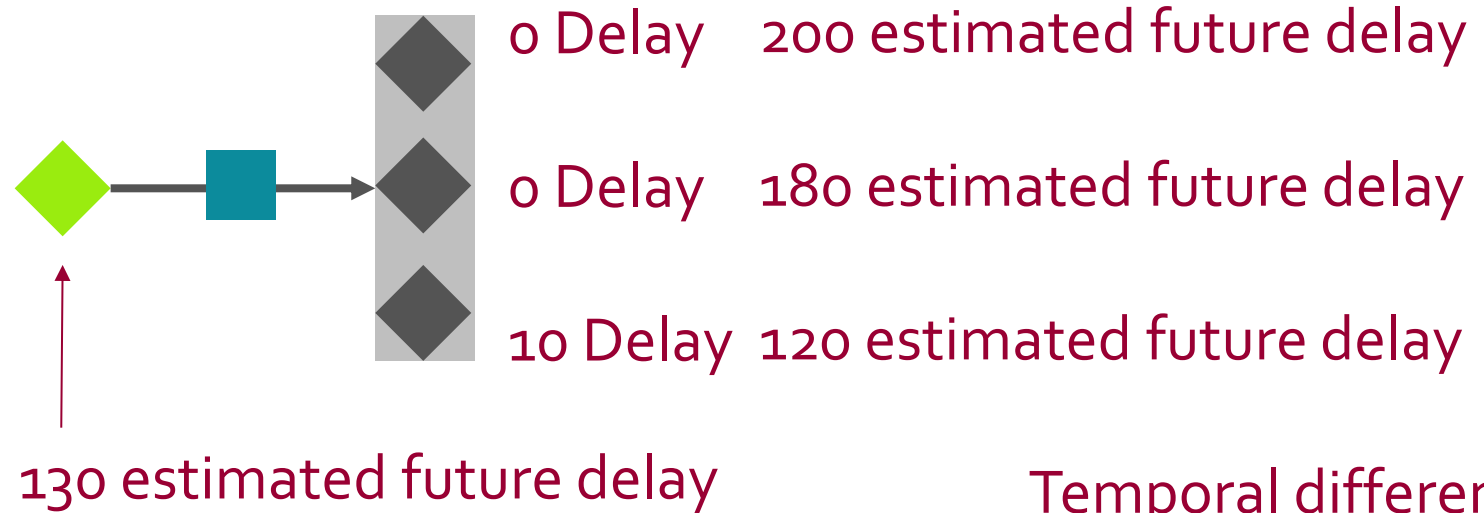
- Approximate the value of being in state  $S$  at time  $t$  as  $\bar{V}_t^{m-1}(S)$ .
- Run a simulation of the vehicles arriving  $(j_0^m, j_1^m, \dots, j_T^m)$
- Compute  $\hat{v}_t^m = \min_{S_{t+1} \in Y(S_t^m, j_t^m)} (C(S_t^m, j_t^m, S_{t+1}) + \bar{V}_{t+1}^{m-1}(S_{t+1}))$
- Set  $\bar{V}_t^m(S_t) = \begin{cases} (1 - \alpha_m)\bar{V}_t^{m-1}(S_t) + \alpha_m \hat{v}_t^m & S_t = S_t^m \\ \bar{V}_t^{m-1}(S_t) & \text{otherwise.} \end{cases}$
- Repeat for  $m = m + 1$

## Temporal differencing

- Define  $\delta_\tau^m = C(S_\tau^m, j_\tau^m, S_{\tau+1}^m) + \bar{V}_{\tau+1}^{m-1}(S_{\tau+1}^m | j_\tau^m) - \bar{V}_\tau^{m-1}(S_\tau^m | j_{\tau-1}^m)$  for  $\tau = t \dots T$
- Instead set  $\bar{V}_t^m(S_t) = \begin{cases} \bar{V}_t^{m-1}(S_t^m) + \alpha_m \sum_{\tau=t}^T \lambda^{T-\tau} \delta_\tau^m & S_t = S_t^m \\ \bar{V}_t^{m-1}(S_t) & \text{otherwise.} \end{cases}$

# Approximate dynamic programming

Simulate the drivers...

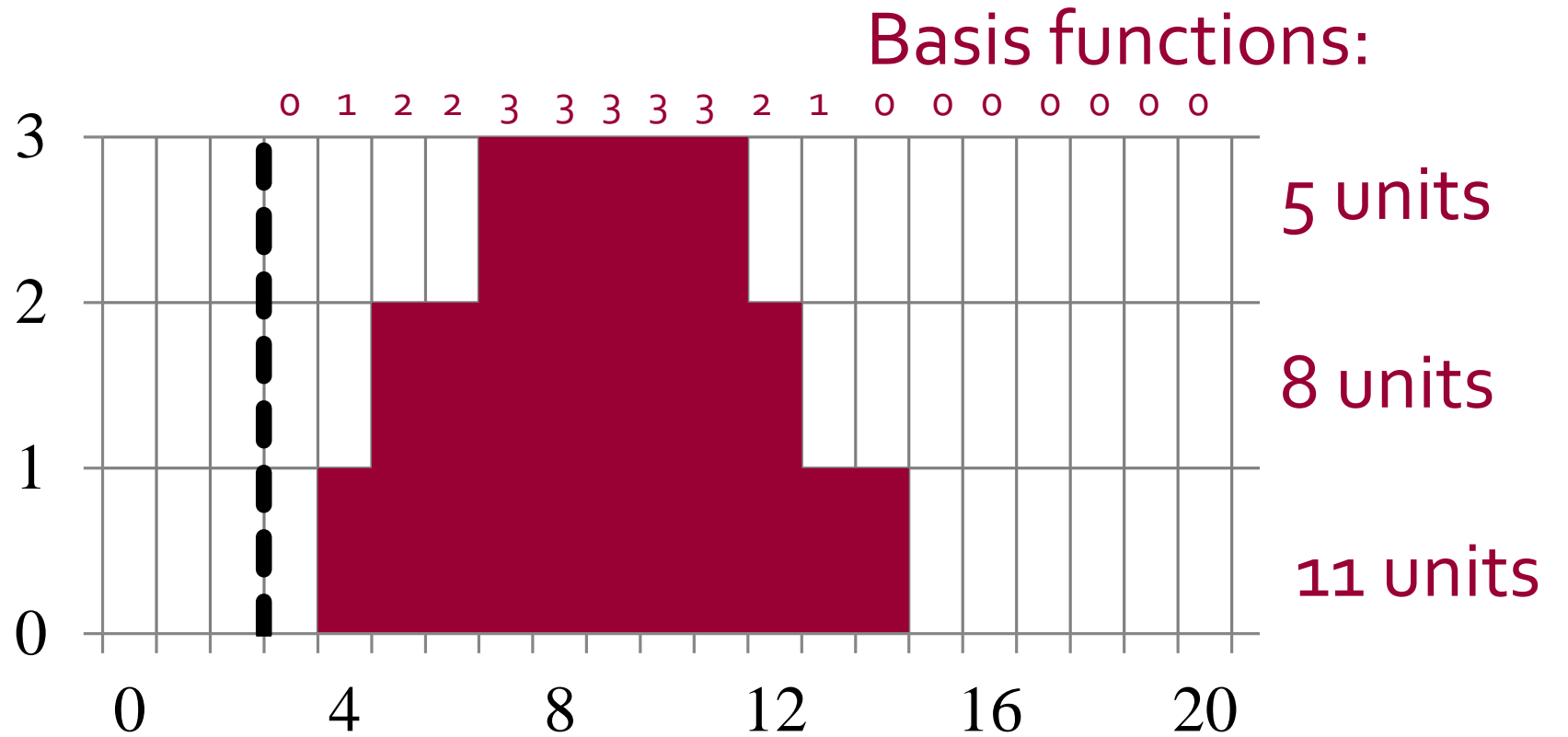


Temporal differencing  
– base estimate on many steps ahead

## Linear value function approximation

- Still need to define  $\bar{V}_t^m(S)$  for each  $S$  (and there are many!)
- Instead let:  $\bar{V}_t^m(S) = \sum_{f \in \mathcal{F}} \theta_{tf}^m \phi_f^m(S)$ , now goal is to find best  $\theta_{tf}^m$
- Approximation functions are:
  - For station  $b_i$  having  $n_i$  batteries: for each value  $q = 1, \dots, n_i$ , the define function  $\phi_{iq}$  which maps state  $S \in \mathcal{S}$  to the number of time periods in which **station  $b_i$  has at least  $q$  batteries reserved.**
- Only need one set of coefficients for all time since basis functions naturally decrease as time progresses, so  $\theta_f^m = \theta_{tf}^m$  for all  $t$

Simplify  
using linear  
functions



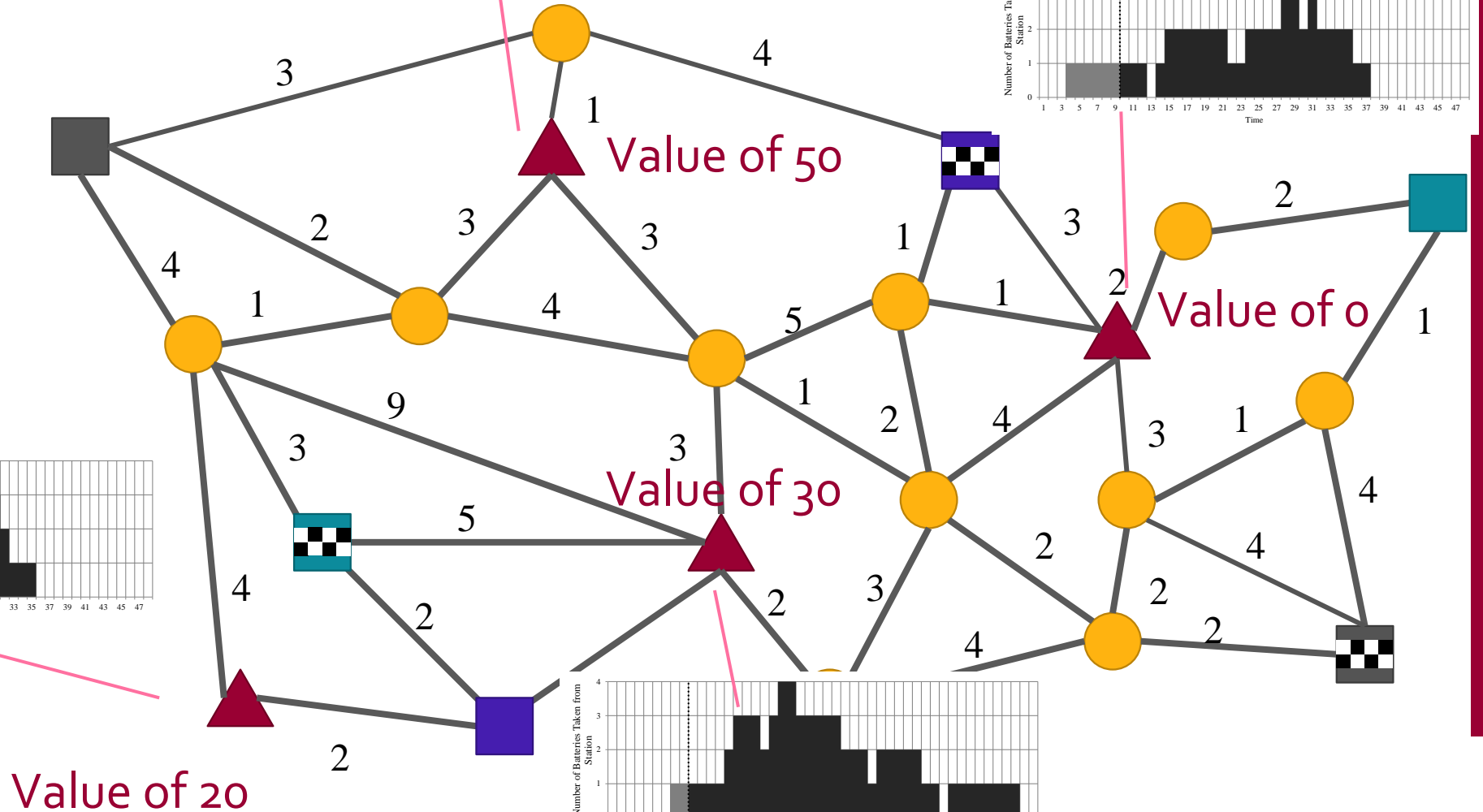
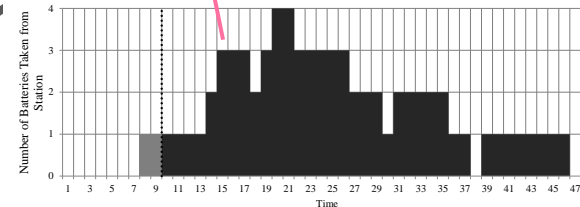
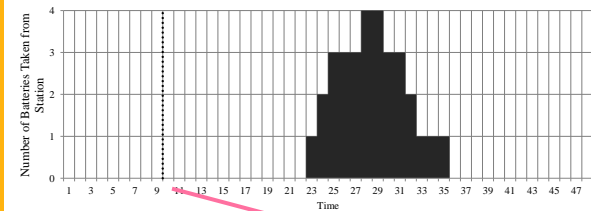
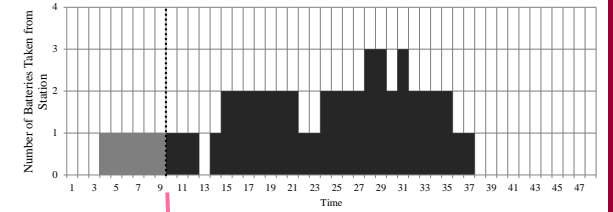
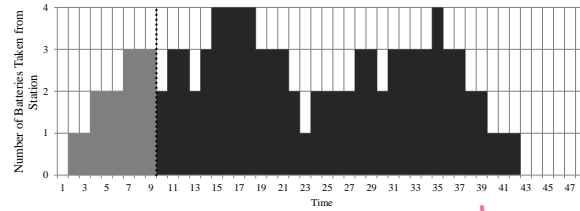
## One last complication

- We know where the costs are being incurred (either from waiting at a particular station or a longer route)
- We don't want stations to be penalized for vehicles waiting at other stations
- So let  $\bar{V}_t^{m,1}(S_t)$  be the cost from a longer path and swap times and let  $\bar{V}_{t,i}^{m,2}(S_t)$  be the cost from having to wait at a particular station  $i$
- $\bar{V}_t^m(S_t) = \bar{V}_t^{m,1}(S_t) + \sum_{i=1}^{\beta} \bar{V}_{t,i}^{m,2}(S_t)$
- And the costs can be calculated directly for more accurate approximations

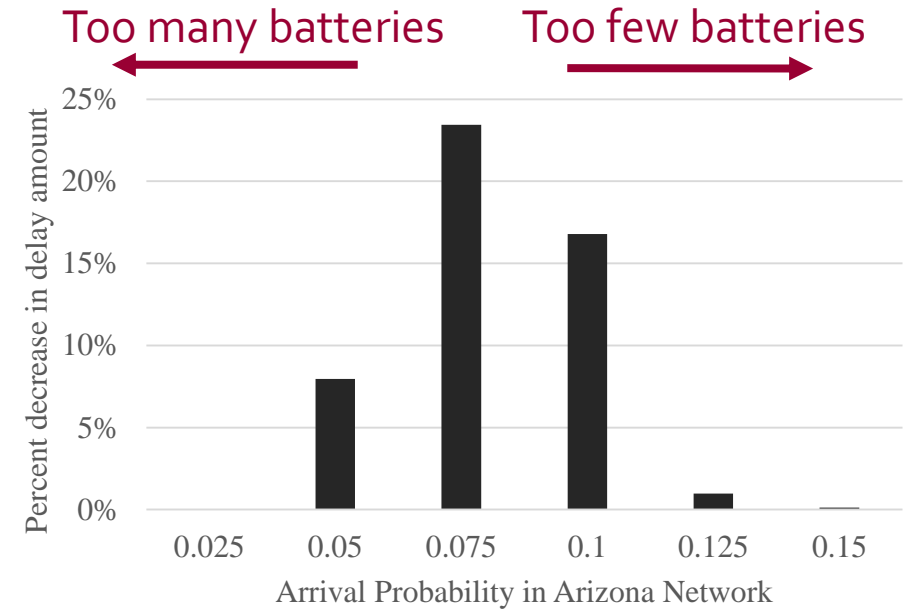
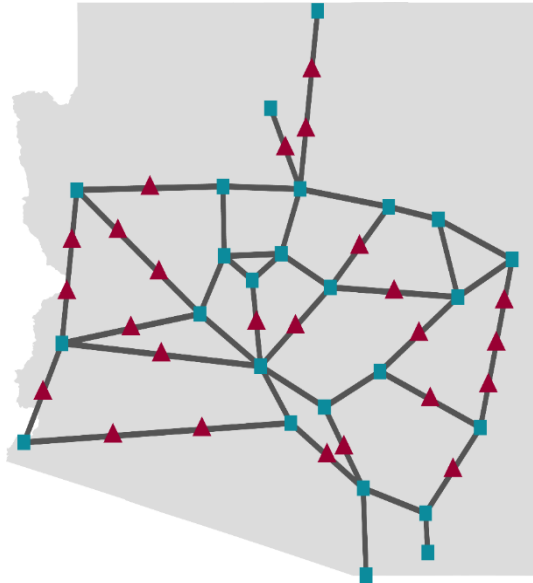


Last complication

The state has a value of 200 time units



# Results tested on Arizona network



- Arizona road network used
- Arrival probability of vehicles adjusted over several runs
- Compared to greedy policy (cars always act in best interest)
- Up to 23% shorter delays on average

# Extensions

- How do you adjust the problem to:
  - Handle non-constant demand throughout the day?
  - Have vehicles drop off batteries that are not empty?
  - Start the route with a not full battery?

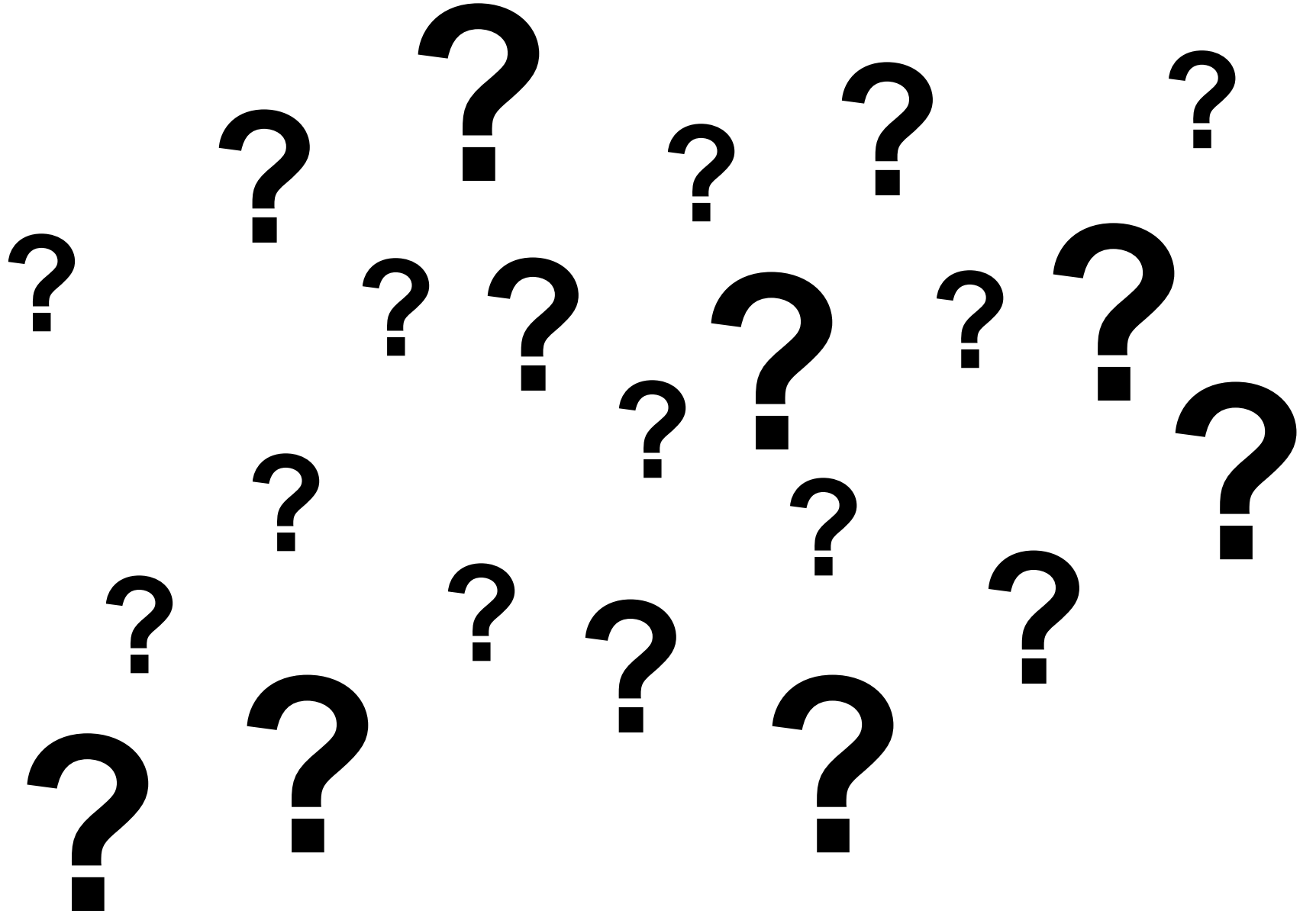
# Contributions

- Formulated a new model for minimizing travel times globally for a set of electric vehicles in a stochastic online setting
- Found a method for finding good policies based on Markov chance-decision processes and approximate dynamic programming
- Tested the method on Arizona highway data and got a 23% improvement compared to greedy routing

# Future work

1. EV shortest walk problem
  - Allow for stochastic cases with learning
  - Improve speed of stochastic case
2. Online EV routing and reservations
  - Add stochastic arc lengths and surprise arrivals
  - Improve ADP results

Questions?



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