Stochastic Service Network Design with Rerouting

Ruibin Bai

University of Nottingham

Ningbo China

ruibin.bai@nottingham.edu.cn

Stein W Wallace

Norwegian School of Economics

stein.wallace@nhh.no

Outline

- Motivation
- Service network design - Previous models
- Stochastic SND with rerouting
- Computational Analyses
- Conclusions

Motivation

- In 2012, Amazon recorded USD 61 billion in sales, a 27.1% increase from 2011.
- Fuelled by massive sales, Taobao.com generated more than USD 3 billion sales on a single day on November 11, 2012, creating 80 million delivery requests.
- The total online shopping sales in 2012 in China were estimated to be USD 1.3 trillion, up 27.9% from 2011 while the total number of deliveries is estimated to be 6 billion (CECRC, 2013)

Freight Service Network Design: A Deterministic Model

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A Deterministic Model (M-Determ)

$$\min \sum_{i \in \mathcal{N}} \sum_{i \in \mathcal{N}} \sum_{t=0}^{T-1} c_{ij} x_{ij}^t$$

s.t.

$$\sum_{j \in \mathcal{N}} x_{ji}^{t^-} = \sum_{j \in \mathcal{N}} x_{ij}^t \quad \forall i, \forall t$$

$$\sum_{k \in \mathcal{K}} y_{ijk}^t \leq u x_{ij}^t \quad \forall (i,j), \forall t, \forall i \neq j$$

$$-\sum_{j \in \mathcal{N}} y_{jik}^{t^-} + \sum_{j \in \mathcal{N}} y_{ijk}^t = \begin{cases} d^k & \text{if } (i,t) \text{ is supply node for } k \\ -d^k & \text{if } (i,t) \text{ is demand node for } k \\ 0 & \text{otherwise} \end{cases}$$

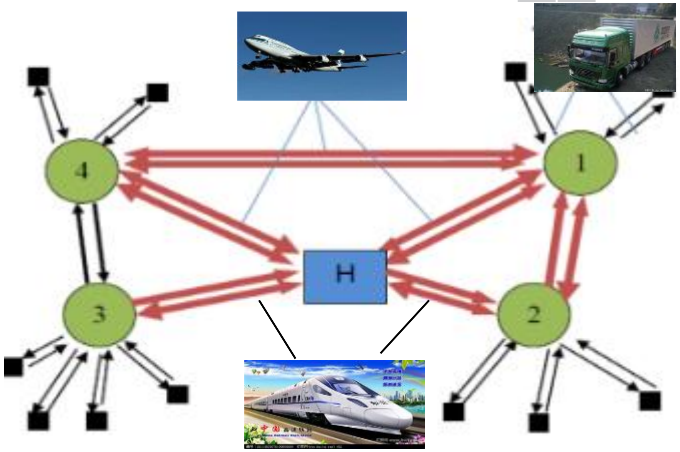
$$y_{ijk}^{\tau(k)} = 0$$

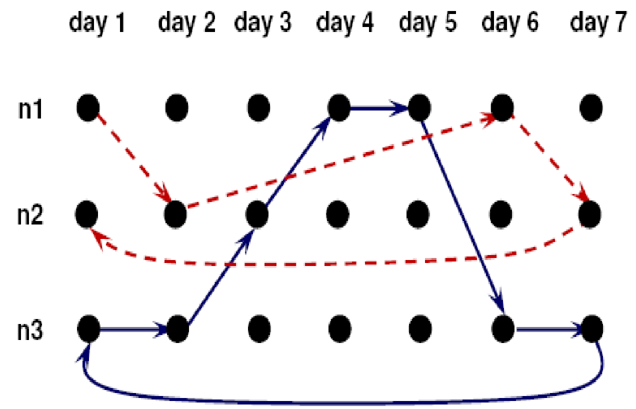
$$x_{ij}^t \in \mathbb{Z}^+$$

$$y_{ijk}^t \geq 0$$

Demand not always deterministic

Decision variables





Time-space network

A Stochastic Model for Service Network Design with Outsourcing

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M-Stoch1:

$$\min\{\mathbf{cx} + \lambda \sum_{s} p^s Q_1(\mathbf{x}, \mathbf{d}^s)\}$$

s.t

$$\sum_{j \in \mathcal{N}} x_{ji}^{t^-} = \sum_{j \in \mathcal{N}} x_{ij}^{t} \quad \forall i, \forall t$$

$$x_{ij}^{t} \in \mathbb{Z}^+ \quad \forall (i,j), \forall t$$

- **Lium, Crainic & Wallace (2009), *Transportation Science* 43:144-157**
- **Demand stochasticity captured by a scenario tree.**
- **A two stage stochastic programming model.**
- **Part of the demand can be outsourced if it is too expensive to serve.**

Sub-Problem

$$Q_1(\mathbf{x}, \mathbf{d}^s) = \min \sum_{k \in \mathcal{K}} Z^s(k)$$

$$\sum_{k \in \mathcal{K}} y_{ijk}^{st} \leq u x_{ij}^{t} \quad \forall i, \forall j, \forall t, \forall i \neq j$$

$$-\sum_{j \in \mathcal{N}} y_{jik}^{st^-} + \sum_{j \in \mathcal{N}} y_{ijk}^{st} = \begin{cases} d_s^k - Z^s(k) & \text{if } (i,t) \text{ is supply node for } k \\ -d_s^k + Z^s(k) & \text{if } (i,t) \text{ is demand node for } k \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ijk}^{s\tau(k)} = 0 \quad \forall i, \forall j, \forall k$$

$$y_{ijk}^{st} \geq 0 \quad \forall i, \forall j, \forall k, \forall t.$$

$$Z^s(k) \geq 0 \quad \forall k.$$

Stochastic Service Network Design with Rerouting

- Justification for rerouting
  - Exists in real-world
  - A flexible approach to deal with various uncertainties and disruptions.
  - Outsourcing is less popular in a competitive market.
- Aims
  - Study how rerouting will impact on the stochastic network design
  - Obtain managerial insights for practical applications.

Stochastic Service Network Design with Rerouting

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- Decision Variables:

Notation	Meaning
x_{ij}^{t}	**The service frequency on arc (*i*,*j*) in period *t*.**
y_{ijk}^{st}	**The flow of commodity *k* on arc (*i, j*) in period *t* and scenario *s*.**
v_{ij}^{st}	**The number of vehicles increased on arc (*i*,*j*) in period *t*, scenario *s***
w_{ij}^{st}	**The number of vehicles decreased on arc (*i*,*j*) in period *t*, scenario *s***
$Z^{s}(k)$	**The amount of outsourcing required for commodity *k* in scenario *s*.**
$\mathbf{y}^{s}$	**Vector of flow variables for scenario *s*.**
$\mathbf{x}, \mathbf{v}^{s}, \mathbf{w}^{s}$	**Vectors for design and rerouting variables.**

Stochastic Service Network Design with Rerouting (M-Stoch2)

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Master Problem

$$\min \; \mathbf{cx} + Q(\mathbf{x})$$

$$\sum_{j\in\mathcal{N}} x_{ji}^{t-} = \sum_{j\in\mathcal{N}} x_{ij}^{t} \quad \forall i, \forall t$$

$$x_{ij}^{t} \in \mathbb{Z}^{+} \quad \forall (i,j), \forall t$$

$$Q(\mathbf{x}) = \sum_{s} p^s Q(\mathbf{x}, \mathbf{d}^s)$$

Sub-Problem

$$Q(\mathbf{x}, \mathbf{d}^s) = \min \{\gamma \mathbf{cv}^s - \eta \mathbf{cw}^s + \lambda \sum_{k\in\mathcal{K}} Z^s(k)\}$$

$$\sum_{j\in\mathcal{N}} (v_{ji}^{st^-} - w_{ji}^{st^-} + x_{ji}^{t^-}) = \sum_{j\in\mathcal{N}} (v_{ij}^{st} - w_{ij}^{st} + x_{ij}^{t}) \quad \textbf{(1)}$$

$$\sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}} (v_{ij}^{st} - w_{ij}^{st}) = 0 \quad \forall t \quad \textbf{(2)}$$

$$w_{ij}^{st} \leq x_{ij}^{t} \quad \forall i, \forall j, \forall t \quad \textbf{(3)}$$

$$\sum y_{ijk}^{st} \leq u(v_{ij}^{st} - w_{ij}^{st} + x_{ij}^{t}) \quad \textbf{(4)}$$

$$-\sum_{j\in\mathcal{N}} y_{jik}^{st^-} + \sum_{j\in\mathcal{N}} y_{ijk}^{st} = \begin{cases} d_s^k - Z^s(k) & \text{if } (i,t) \text{ is supply node for } k \\ -d_s^k + Z^s(k) & \text{if } (i,t) \text{ is demand node for } k \\ 0 & \text{otherwise} \end{cases} \quad \textbf{(5)}$$

$$y_{ijk}^{s\tau(k)} = 0 \quad \forall i, j, k \quad \textbf{(6)}$$

$$v_{ij}^{st}, w_{ij}^{st} \in \mathbb{Z}^{+} \quad y_{ijk}^{st} \geq 0 \quad Z^s(k) \geq 0 \quad \textbf{(7)}$$

Experimental settings

- All the models were solved by the Cplex12 callable library.
- Running on a PC with 2.8GHz Intel i7 CPU and 4.0GB RAM
- Random demand: Tri(5,11,8).
- Correlation matrices: uncorrelated (0), positive (0.7) and mixture of positive (0.4) and negative (-0.5).
- Scenario generator: Høyland, Kaut and Wallace, COAP (2003).

fixed costs matrix (c_{ij})					
100	150	150	250	250	250
150	100	150	250	250	250
150	150	100	250	250	250
250	250	250	100	150	150
250	250	250	150	100	150
250	250	250	150	150	100

parameters	**values**
$\|\mathcal{N}\|$	6
$\|\mathcal{K}\|$	8
T	5
u	20
λ	150
γ	1.05
η	0.95
no. of scenarios	20
p^s	1/20

A General Comparison of Different Models

Models/Evaluation	Meaning
M-Determ (or Determ-Stoch1)	The deterministic service network, evaluated in stage 2 of M-Stoch1.
M-Stoch1	The stochastic service network by Lium, Crainic and Wallace (2009).
M-Stoch2	Our proposed model – with rerouting option
Determ-Stoch2	The deterministic service network, evaluated in stage 2 of M-Stoch2.
Stoch1-Stoch2	The stochastic service network by M-Stoch1, evaluated in stage 2 of M-Stoch2.

Small cases

Corr. type	M-Stoch2		M-Determ			M-Stoch1		
	obj	$\sum z$	obj	Loss %	$\sum z$	obj	Loss %	$\sum z$
0	2514	4.0	2549	1.4	10.3	2547	1.3	9.1
+	2581	10.5	2664	3.2	24.3	2648	2.6	21.6
+/-	2561	8.7	2621	2.4	17.8	2612	2.0	16.9

8 commodities
6 nodes
5 days
20 scenarios

Small cases

Corr type	Determ-Stoch2			Stoch1-Stoch2		
	objective	Loss%	$\sum z$	objective	Loss%	$\sum z$
0	2515	0.02	4.1	2523	0.34	1.8
+	2582	0.03	10.8	2594	0.52	3.9
+/-	2563	0.09	8.7	2571	0.41	10.5

M-Determ	0.2
M-Stoch1	1.0
M-Stoch2	884.6
Determ-Stoch2	3.3
Stoch1-Stoch2	3.4

CPU times seconds

For larger cases – 20 commodities

- Though Determ-Stoch1 is quite a bit weaker than M-Stoch1 (VSS around 8-10%),
  - M-Determ is often better than M-Stoch1 in the rerouting model.
  - In our tests M-Determ is a bit better on average.

Averages	objective	$\sum z$	CPU seconds
Determ-Stoch2	3694	3.2	1790
Stoch1-Stoch2	3710	3.3	460
M-Stoch2	3775	7.4	> 4 hours

Solution Structural Differences

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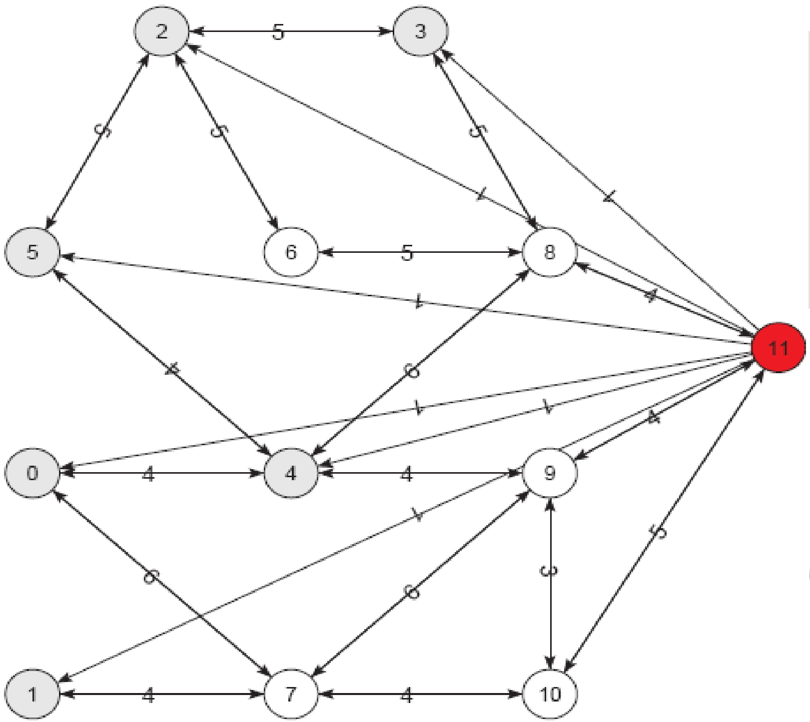
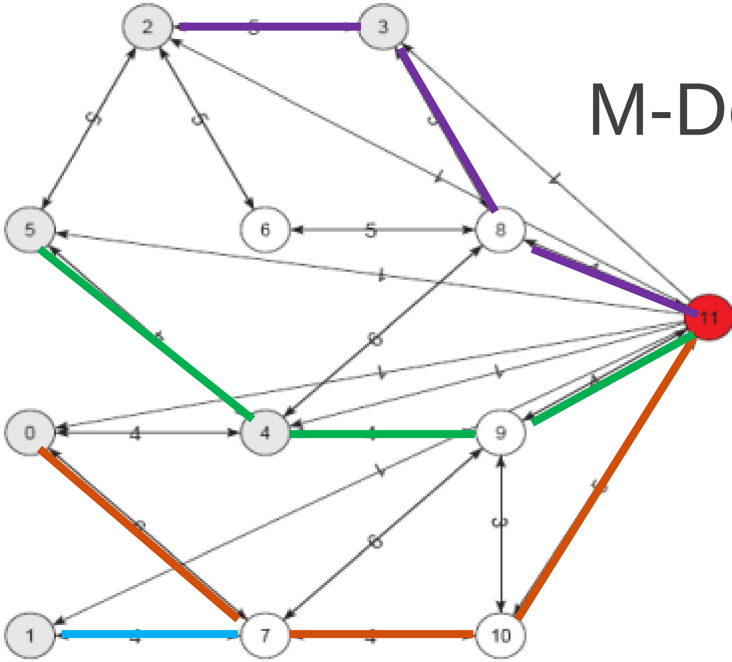


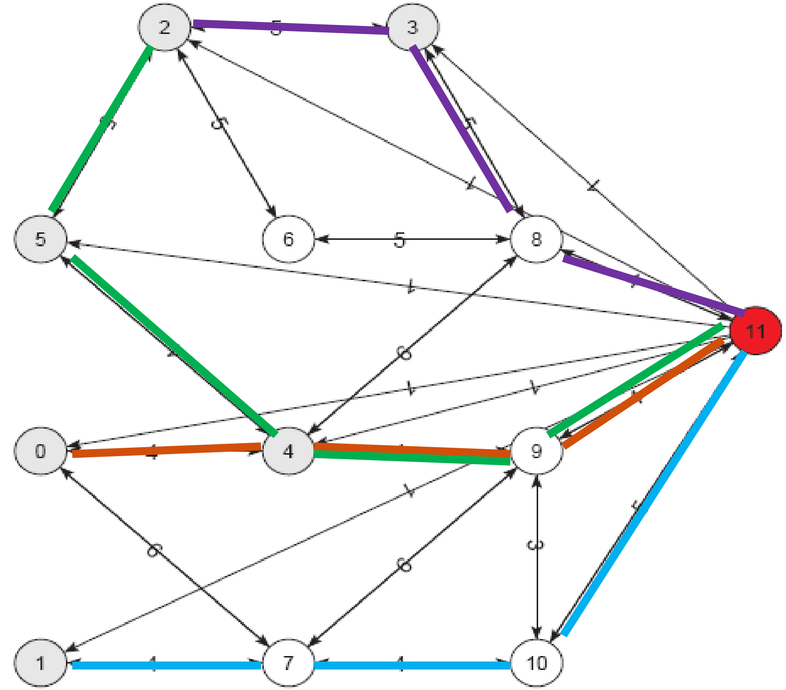
- Nodes 0-5 are sources and node 11 is destination node.
- The demand distribution Tri(0, 1, 0.5)
- Service capacity is 1 unit.

Table 6: Correlation matrix for the instance LTL6-SW

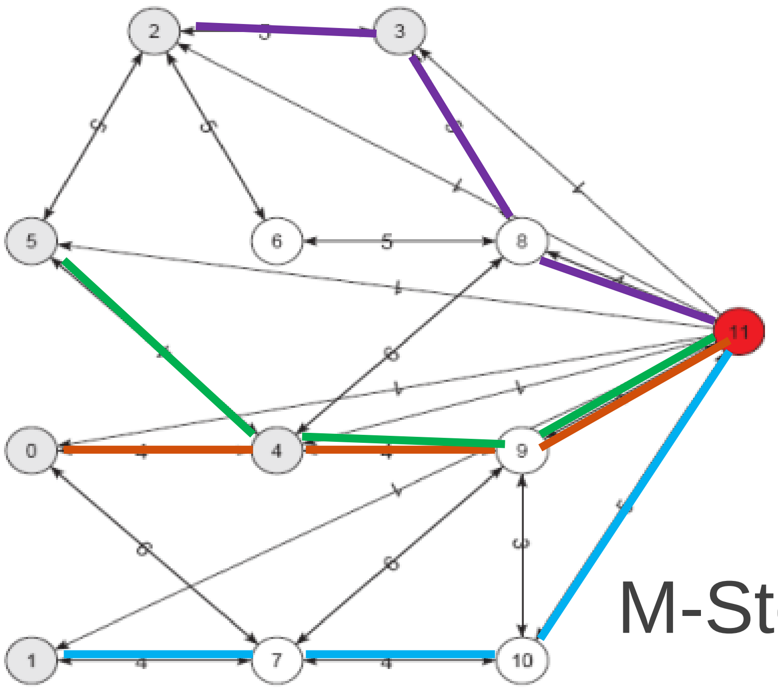
	n0	n1	n2	n3	n4	n5
n0	1	0.7	0.4	0.4	-0.7	-0.7
n1	0.7	1	0.4	0.4	-0.7	-0.7
n2	0.4	0.4	1	0.8	-0.5	-0.5
n3	0.4	0.4	0.8	1	-0.5	-0.5
n4	-0.7	-0.7	-0.5	-0.5	1	0.7
n5	-0.7	-0.7	-0.5	-0.5	0.7	1



M-Determ



M-Stoch1



M-Stoch2

Table 6: Correlation matrix for the instance LTL6-SW

	n0	n1	n2	n3	n4	n5
n0	1	0.7	0.4	0.4	-0.7	-0.7
n1	0.7	1	0.4	0.4	-0.7	-0.7
n2	0.4	0.4	1	0.8	-0.5	-0.5
n3	0.4	0.4	0.8	1	-0.5	-0.5
n4	-0.7	-0.7	-0.5	-0.5	1	0.7
n5	-0.7	-0.7	-0.5	-0.5	0.7	1

Rerouting versus Outsourcing

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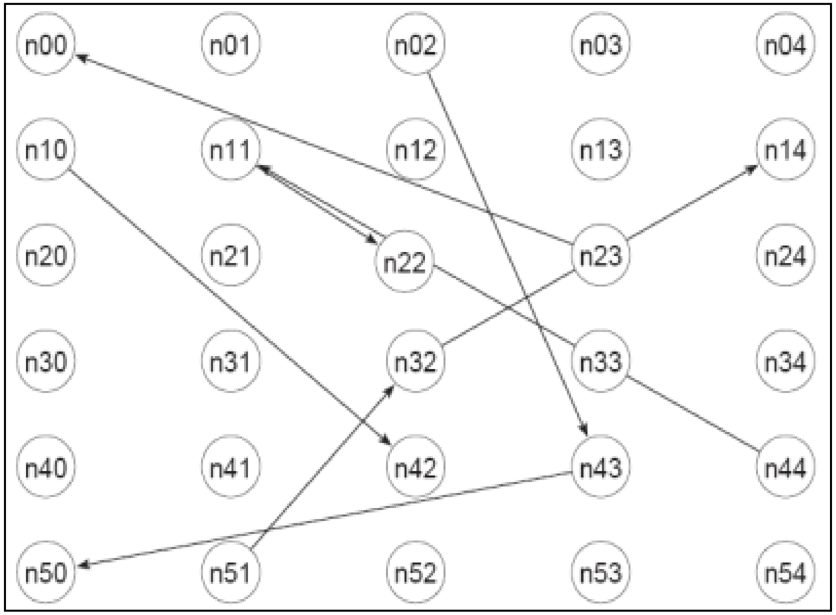
How does M-Stoch2 balance between rerouting and outsourcing?

- by changing parameters *γ*, *η*

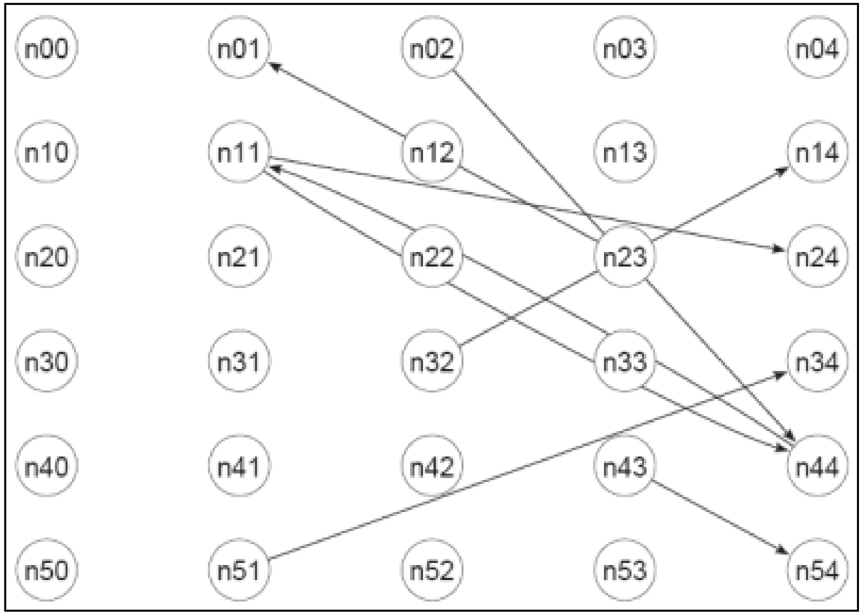
	Determ-Stoch2		Stoch1-Stoch2		M-Stoch2	
(γ, η)	obj	$\sum Z$	obj	$\sum Z$	obj	$\sum Z$
(1.05,0.95)	**3432.9**	8.5	3448.6	8.0	**3432.9**	8.5
(1.0,0.9)	3493.9	16.2	3507.3	8.0	**3485.5**	8.0
(1.125,0.875)	3514.7	17.4	3516.4	8.0	**3507.7**	16.2
(1.175,0.825)	3551.0	17.4	**3531.3**	8.0	**3531.3**	8.0
(1.25,0.75)	3598.4	23.4	**3539.7**	12.1	**3539.7**	12.1

Impact of the commodities' spatial-temporal distribution

Exactly the same commodity sets (O-D pairs)



Balanced



Clustered

Impact of the commodities' spatial-temporal distribution

- The expected costs are about 10% lower in the clustered case for all models.
- It is more important to include rerouting in the clustered case (gains are 3 times larger in our tests).

- Clustering gives more opportunity for consolidation and more value to possible rerouting.

Conclusions and Future Research

- **Rerouting is an effective way to tackle uncertainty. It tends to outsource considerably less than the existing approaches.**
- **When the rerouting cost is moderate, experimental results suggest that the deterministic solution may not be as “brittle" as was previous thought.**
- **For large instances M-Stoch2 is generally unsolvable. Decomposition-like heuristics in the forms of Determ-Stoch2 and Stoch1-Stoch2 are promising.**
- **When demand is highly uncertain and correlated (both positive and mixed), the savings made through robust network design are among the highest.**
- **The spatial-temporal distribution of demands could have a big impact on profitability. The implication for freight companies is to develop a market with certain beneficial spatial-temporal characteristics.**

Bai, Wallace, Li, Chong, Stochastic service Network design with rerouting. *Transportation Research Part B* 60 (2014) 50-65.