Solving the vehicle routing problem with stochastic demands by a branch-and-cut-and-price algorithm

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The vehicle routing problem with stochastic demands (VRPSD) is defined on a graph $G = (V, E)$. The nodes V of the graph consist of a depot node (node 0) and n customer nodes $V_c = \{1, \ldots, n\}$. The demand of each customer $i \in V_c$ is given by a stochastic variable ξ_i with known distribution. It is assumed that the stochastic variables are independent and follow the same distribution, but that the parameters describing the distributions may be different. In the following we will think of the customer demand as an amount of goods that we need to pick up. The customers are served by a homogeneous fleet of vehicles with capacity Q. It is typically assumed that all routes must satisfy that the total expected demand of the customers on the route is less than Q. The VRPSD is formulated as a two-stage stochastic recourse problem. In the first stage we plan routes for the vehicles without knowing the actual demands. In the second stage we carry out the plan and the actual demands are revealed. During stage two we may encounter routes where we reach a customer and are unable to pick up his full demand because the capacity of the vehicle is reached. In that case we will resort to a simple recourse action which consists of filling up the vehicle at the customer, driving back to the depot to empty the vehicle, returning to the customer to pick up the remaining load and then continue on the route. The objective of the VRPSD is to design the routes in stage 1 such as to minimize the expected cost (e.g. distance driven) of the stage 2 solution.

The VRPSD has received considerable attention in the literature, but far less than its deterministic sibling, the capacitated vehicle routing problem (CVRP). State of the art exact methods for solving the VRPSD are proposed in Laporte et al. [2002], Christiansen and Lysgaard [2007] and Jabali et al. [2012]. The methods proposed by Laporte et al. [2002] and Jabali et al. [2012] use branch-and-cut algorithms based on the integer L-shaped algorithm while the method proposed by Christiansen and Lysgaard [2007] use branch-and-price. In this talk we follow the branch-and-price approach. The VRPSD is formulated as a set partitioning problem:

$$\min \sum_{p \in \Omega} c_p x_p$$

subject to

$$\sum_{p \in \Omega} a_{ip} x_p = 1 \qquad \forall i \in V_c$$

$$x_p \in \{0, 1\} \quad \forall p \in \Omega$$

where Ω is the set of all feasible routes, x_p is a binary decision variable that indicates if route $p \in \Omega$ should be included in the solution, c_p is the expected cost of route p and a_{ip} is a parameter that indicates if customer $i \in V_c$ is served on route $p \in \Omega$. Since the model quickly grows very large its linear programming relaxation is solved by column generation and the resulting lower is used in a branch-and-price algorithm. In the column generation algorithm we need to generate routes that have negative reduced cost given a current set of dual variables. Note that the computation of the reduced cost of a route also includes the computation of the expected cost of the route. Christiansen and Lysgaard [2007] proposed to solve a relaxed version this problem using an ordinary shortest path algorithm in a expanded network. We show how this approach can be translated into an ordinary labeling algorithm for a resource constrained shortest path problem. We show that the resulting dominance criterion can be significantly improved for stochastic demands that follow a Poisson distribution (the case considered in Christiansen and Lysgaard [2007]). Furthermore we improve the LP relaxation by using the concept of ng-routes proposed by Baldacci et al. [2011] that imply that the routes returned by the pricing sub-problem are close to being elementary. On top of this we add valid inequalities known for the CVRP to further improve the LP relaxation of the set partitioning formulation. We use the CVRPSEP package by Lysgaard [2003] for this purpose.

Christiansen and Lysgaard [2007] tested their algorithm on 40 instances derived from standard CVRP instances and were able to solve 19 of these. The algorithm presented in this talk solves 39 of the 40 instances (albeit on a more modern computer and allowing more computing time). These results and results on a wider set of instances are presented in the talk.

References

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