A BRANCH-AND-PRICE ALGORITHM FOR THE VEHICLE ROUTING PROBLEM WITH TIME WINDOWS CONSIDERING DRIVING AND WORKING HOUR REGULATIONS

S. DABIA   E. DEMIR   N. P. DELLAERT   T. VAN WOENSEL   M. STEADIESEIFI
SCHOOL OF INDUSTRIAL ENGINEERING, EINDHOVEN UNIVERSITY OF TECHNOLOGY, THE NETHERLANDS

1. Introduction

VRPTW have been widely studied, applied in practice, and further developed to include more realistic constraints. Gendreau and Taratilis [4] and Baldacci *et al.* [1] are two of the most recent surveys on the VRPTW. One of the recent series of constraints is the series of restrictions on the amount of driving and working hours of truck drivers. During the last decade, many legislation bodies around the world have passed particular bills concerning driving and working hours of drivers, which both drivers and liable companies are compelled to follow. The European Union Regulation (EC) No. 561/2006 and Directive 2002/15/EC are examples of these rules which have been enforced since 2007. Prescott-Gagnon *et al.* [14] and Drexl and Prescott-Gagnon [3] have cearly explained these regulations. In real-life, solutions derived from VRPTW problems excluding breaks and rests could be infeasible and result in driver's exhaustion and probably accidents. On the other hand, if these breaks and rest are applied to the solution of the classic VRPTW in a post-processing step, drivers could arrive at the customers after inevitable delays. As a result, in order to guarantee the feasibility, these restrictions should be included in the VRPTW modelling. To the best of our knowledge, this new problem which now on in this paper is called *VRPTW-DWR*, has recently attracted the attention of researchers but no exact solution method has been proposed to solve VRPTW-DWR. This paper provides a mathematical programming formulation and optimally solves this model using a tailored Branch-and-Price (B&P) algorithm. In a branch-and-price algorithm, the linear relaxation in each branch-and-bound node is solved using column generation and the pricing problem is an elementary shortest path problem with resource constraints (ESPPRC). This pricing problem is then solved by means of a labeling algorithm which generates columns with negative reduced cost. To improve the performance of the labeling algorithm, new dominance criteria are introduced to eliminate labels that are not leading the routes in the optimal solution. To speed up the branch-and-price algorithm, two heuristics are designed to find the columns with negative reduced cost. Although the shortest path problem here results in worse lower bounds, it is easier to solve and the integrality of the master problem is still guaranteed by branch-and-bound.

2. Driving and Working Hour Regulations in Routing

The VRPTW-DWR has mainly attracted the attention of researchers during the last a few years. The work of Savelsbergh and Sol [15] seems to be the first paper to consider lunch breaks and night rests in the vehicle routing and scheduling problem with pickup and delivery in a fixed time interval, and to propose a branch-and-price algorithm. Zäpfel and Bögl [18] proposed a two-phased heuristic algorithm for a VRPTW where in the first phase it solves a vehicle routing problem with daily breaks and maximum time interval, and in the second phase it tackles driver's rests and weekly time intervals as a personnel assignment problem.

Xu *et al.* [17] have been the first to include driving regulations and night rests enforced by the government (in the US). The authors considered these regulations in a rich pickup and delivery problem with multiple time windows, and argued that its complexity is NP-hard. They also proposed a column generation heuristic algorithm to solve their routing problem.

Goel [5] and [6] applied the EU regulations into a VRPTW and designed a large neighborhood search heuristic in which a labeling algorithm was used to check the feasibility of a solution at each removal and insertion. Another example of using labeling concept in a large neighborhood search is the work of Prescott-Gagnon *et al.* [14] in which a tabu search column generator was used to construct new solutions.

Kok *et al.* [10] used a restricted dynamic programming heuristic for the VRPTW-DWR. This work was the first paper to include all of the EU driving and working regulations including standard and extended. Moreover, the authors showed that their algorithm works for longer time horizons and even rolling horizon frameworks. Kok *et al.* [9], in a post-processing departure time optimization of VRPTW problems, included break and rest scheduling, modeled it as an ILP problem and solved it to optimality.

Meyer *et al.* [13] and Meyer [12] also applied this restricted dynamic programming algorithm for the case of distributed decision making where companies make only routing decisions and leave the break scheduling decisions to the drivers.

Goel and Kok [8] studied vehicle scheduling problems with time windows and team drivers where one driver can drive and the other can rest for 4.5 hours. They proposed a depth-first-breath-second search algorithm with dominance.

Derigs *et al.* [2] introduced EU regulations in a multi-trip VRP, in a air cargo road feeder service business, and presented a neighborhood search algorithm with a decomposition approach where the neighborhood search generates the trips, then a packing heuristic aggregates them to multi-trips.

Finally, the recent work of Goel *et al.* [7] is a study of similar laws valid in Australia for drivers' fatigue, in which they have designed a dynamic programming heuristic algorithm as their solution method.

3. Modeling framework

The research of Kopfer and Meyer [11] seems to be the first paper to model European Union driving regulations as an integer linear programming (ILP) in the form of Traveling Salesman Problem with Time Windows (TSPTW). Zäpfel and Bögl [18] proposed an ILP formulation for their multi-period vehicle routing and scheduling problem where breaks are considered and driving jobs can be outsourced. The work of Wen *et al.* [16] is another example of modeling a vehicle routing and scheduling problem with time windows and standard breaks for both internal and external drivers of vehicles. Goel *et al.* [7] designed a state-space scheduling model for their VRPTW including Australian laws. Meyer M2011 also modeled VRPTW with the EU regulations as a state-space scheduling model. Goel and Kok [8] is another example which applied state-space scheduling modeling for the case of team drivers.

4. Solution methodology

The VRPTW-DWR is reformulated as a set-partitioning problem (the master problem), where in the set P of all feasible paths, the goal is to find the path with minimum travel cost. A feasible path p is defined by the sequence of customers visited along it, while respecting the capacity constraints, time windows, and break scheduling constraints. For each path $p \in P$, c_p is defined as the cost of the path, the constant σ_{ip} measures the number of times customer i is visited by the path p, and y_p is the binary variable that gets value 1 if and only if the path p is included in the solution.

For the pricing problem, column generation algorithm is embedded in a branch-and-bound framework to guarantee integrality. First, it branches on the aggregated variables x_{ij}. Among the fractional variables, it searches for pairs $(i, j), i, j \in V_c$ such that $x^*_{ij} + x^*_{ji}$ is close to 0.5 and puts the branch on the that aggregated variable. If the aggregated variables for all pairs $(i, j), i, j \in V_c$ are integer, it searches for fractional arcs in the current solution and branches on that variable. Then, for each branch candidate, the lower bound in the two offsprings are estimated by solving a LP relaxation using a quick heuristic. Finally, the algorithm applies branch where the lower bound of the child nodes are maximized. However, there is a limit on the breath and depth for the branching. In this paper, the pricing problem is solved by means of a labeling algorithm. This labeling algorithm generates columns with negative reduced cost and the best break schedule. A bi-directional search is performed where the labels are extended in both forward direction (from v_0 to its successors) and backward direction (from v_{n+1} to its predecessors). These forward and backward label extensions are bounded to a specific time t_m in the middle of planning horizon, and later they are merged to construct the complete path. This strategy avoids generating long labels. New dominance criteria are composed in order to eliminate labels that are not going to lead to the optimal solution.

Numerical results and benchmarks against the proposed heuristics in the literature will be presented on the conference.

References

[1] Baldacci R., Mingozzi A., Roberti R. (2012) Recent exact algorithms for solving the vehicle routing problem under capacity and time window constraints. *European Journal of Operational Research* 218: 1–6.

[2] Derigs U., Kurowsky R., Vogel U. (2011) Solving a real-world vehicle routing problem with multiple use of tractors and trailers and EU-regulations for drivers arising in air cargo road feeder services, *European Journal of Operational Research* 213: 309–319.

[3] Drexl M., Prescott-Gagnon E. (2010) Labelling algorithms for the elementary shortest path problem with resource constraints considering EU drivers' rules, *Logistics Research* 2: 79–96.

[4] Gendreau M., Tarantilis C.D. (2010) Solving large-scale vehicle routing problems with time windows: The state of the art. Tech. Rep. 2010-04, CIRRELT.

[5] Goel A. (2009) Vehicle Scheduling and Routing with Drivers' Working Hours, *Transportation Science* 43: 17–26.
[6] Goel A. (2010) Truck Driver Scheduling in the European Union, *Transportation Science* 44(4): 429–441.
[7] Goel A., Archetti C., Savelsbergh M. (2011) Truck driver scheduling in Australia, *Computers and Operations Research* x: xxx–xxx.
[8] Goel A., Kok A.L. (2011) Efficient scheduling of team truck drivers in the European Union, *Flexible Services and Manufacturing Journal* x: xxx–xxx.
[9] Kok A.L., Hans E.W., Schutten J.M.J. (2011) Optimizing departure times in vehicle routing, *European Journal of Operational Research* 210: 579–587.
[10] Kok A.L., Meyer C.M., Kopfer H., Schutten J.M.J. (2010) A dynamic programming heuristic for the vehicle routing problem with time windows and European Community social legislation, *TransportationScience* 44(4): 442–454.
[11] Kopfer H., Meyer C.M. (2009) A model for the traveling salesman problem including the EC regulations on driving hours. In: Fleischmann B., Borgwardt K.H., Klein R., Tuma A. (edts.), "Operations Research Proceedings 2008", Berlin, 289–294.
[12] Meyer C.M. (2011) "Vehicle Routing under Consideration of Driving and Working Hours: A Distributed Decision Making Perspective", Springer Fachmedien Wiesbaden GmbH, Bremen.
[13] Meyer C.M., Kopfer H., Kok A.L., Schutten M. (2011) Distributed Decision Making in Combined Vehicle Routing and Break Scheduling, In: "Second International Conference of Dynamics in Logistics Proceedings 2009", Bremen, 125–133.
[14] Prescott-Gagnon E., Desaulniers G., Drexl M., Rousseau L.M. (2010) European Driver Rules in Vehicle Routing with Time Windows, *TransportationScience* 44(4): 455–473.
[15] Savelsbergh M., Sol M. (1998) Drive: Dynamic Routing of Independent Vehicles, *Operations Research* 46(4): 474–490.
[16] Wen M., Krapper E., Larsen J., Stidsen T.K. (2009) A multi-level variable neighborhood search heuristic for a practical vehicle routing and driver scheduling problem, Working paper of Department of Management Engineering, Technical University of Denmark.
[17] Xu H., Chen Z.L., Rajagopal S., Arunapuram S. (2003) Solving a practical pickup and delivery problem, *Transportation Science* 37(3): 347–364.
[18] Zäpfel G., Bögl M. (2008) Multi-period vehicle routing and crew scheduling with outsourcing options, *International Journal of Production Economics* 113(2): 980–996.