On the Fixed Charge Transportation Problem

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The *Fixed Charge Transportation Problem* (FCTP) is a generalization of the well-known *Transportation Problem*, where the cost for sending goods from origins to destinations is composed of a fixed cost and a continuous cost proportional to the amount of goods sent.

The FCTP is a special case of the *Single Commodity Uncapacitated Fixed Charge Network Flow Problem* (see [Ortega and Wolsey 2003]), which itself is a special case of the more general *Fixed Charge Problem* formulated by [Hirsch and Dantzig (1968)].

In practical applications, the fixed costs may represent toll charges on highways, landing fees at airports, setup costs in production systems, or the cost for building roads. Not only does the FCTP arise in distribution, transportation, scheduling, and location systems, but also in allocation of launch vehicles to space missions, solid-waste management, process selection, and teacher assignment.

Most of the exact methods proposed up to 2012 are based on a textbook mixed-integer programming formulation having continuous variables that represent the flow from origins to destinations and binary variables that model the usage the links between origins and destinations. [Agarwal and Anjea (2012)] studied the structure of the projection polyhedron of such a formulation in the space its binary variables. They developed several classes of valid inequalities, which generalize the set covering inequalities, and derived conditions under which such inequalities are facet defining. Their exact method could solve randomly generated instances with up to 15 sources and 15 sinks.

[Roberti, Bartolini and Mingozzi (2014)] introduced an integer programming formulation with exponentially many variables corresponding to all possible flow patterns from origins to destinations. They showed that the linear relaxation of this formulation is tighter than that of the standard mixed integer programming formulation. They also described different classes of valid inequalities and a column generation method to compute a valid lower bound embedded into an exact branch-and-price algorithm. Computational results showed that the proposed algorithm could solve instances with up to 70 sources and 70 sinks and outperformed the previous exact algorithms from the literature.

In this talk, we describe a new formulation of the problem that enhances the formulation proposed in [Roberti, Bartolini and Mingozzi (2014)]. This new formulation has exponentially many variables and a pseudo-polynomial number of constraints. Columns represent either flow patterns from origins to destinations or flow patterns from destinations to origins. Those two types of patterns are matched together through the constraints in order to have a valid formulation of the problem.

We show that strong lower bounds can be achieved by solving the linear relaxation of the new formulation with column generation and by adding a small subset of constraints. This

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new bounds are, on average, better than the lower bounds provided by the formulation of [Roberti, Bartolini and Mingozzi (2014)]. The new bound is embedded into an exact branch-and-cut-and-price algorithm to achieve an optimal integer solution.

Computational results on benchmark instances from the literature show that the proposed exact algorithm outperforms the previous exact algorithms from the literature as well as the method of [Roberti, Bartolini and Mingozzi (2014)]. It is several time faster and can solve all instances unsolved by [Roberti, Bartolini and Mingozzi (2014)]. Moreover, it can solve much harder FCTP instances with up to 100 origins and 100 destinations in reasonable computing times.

References

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