A New Exact Approach for the Vehicle Routing Problem with Intermediate Replenishment Facilities

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1 Introduction

We present a new exact approach to the problem known as the Vehicle Routing Problem with Intermediate Replenishment Facilities (VRPIRF). VRPIRF is defined on an oriented graph $G = (V, A)$ where the node set $V = \{0, 1 \dots n, n+1 \dots n+p\}$ consists of a *depot* 0, n *clients* and p *facilities*. As in the Capacitated VRP (CVRP), the aim is to find the least cost set of *routes* that visit each client exactly once, the cost of a route being the sum of the costs of the visited arcs. Each client has a *demand* and can be served by one of the homogeneous, fixed capacity *vehicles* based at the depot. The additional feature of VRPIRF is that vehicles are not compelled to go back to the depot after a route but can replenish at facilities and therefore perform a sequence of routes called a *rotation*. However, the rotation of a vehicle must start and end at the depot and its total duration (the sum of the travel times, the service times, and the replenishment times associated with the visited arcs, clients, and facilities, respectively) must not exceed a given *shift duration*.

2 Positioning in the Literature

VRPIRF is a special case of the Multiple Depot VRP with Inter-depot routes (MDVRPI), which generalizes the well-known Multi-Depot VRP problem (MDVRP). MDVRPI presents several depots, each one acting both as the base of its own fleet of vehicles, and as a replenishment facility for vehicles based at other depots. A reference work on MDVRPI is [1]. The authors, which proposed a tabu-search based heuristic, focused on the case with one central depot and p facilities – which we studied. The name of VRPIRF for such variant has been introduced by the authors of [2] : they proposed a local- and tabu-search-based meta-heuristic algorithm. The authors of [3] studied a variant of VRPIRF with additional constraints on replenishment operations, proposing a Branch&Cut algorithm. Another problem closely related to the VRPIRF is the Multi-Trip VRP, in which there are no facilities and the central depot act also as a replenishment facility. Recently, [4] has proposed a memetic algorithm, while [5] designed an exact algorithm.

3 The proposed Algorithm

We propose a new Branch&Cut algorithm for solving the VRPIRF, a problem for which –to the best of our knowledge– not much has been explored for what concerns exact methods. The

B&C method is based on a MILP two-index compact formulation, which makes use of *replenishment arcs* to model the stop to replenish at a facility between two clients. Replenishment arcs are a powerful modeling concept which has been used, as far as we know, by few works in the literature. The paper we refer to is [7], which makes use of such concept in a generalization of the Shortest Path Problem with Resource Constraint (SPPRC) called the Weight Constrained Shortest Path Problem with Replenishment (WCSPP-R) to model activities that reset a cumulated amount of a given resource. The respect of capacity constraints is ensured by separating Capacity Cuts on a first transformation of the graph. Such separation is performed by means of Lysgaard's CVRPSEP routines (see [6]). To impose that a solution is connected, i.e. that the routes of a vehicles actually form a non-discontinuous rotation, we take advantage of replenishment arcs, which allow to represent a rotation as an elementary closed path with both its endpoints on the depot. Connectivity is thus ensured by separation of SECs on a second transformation of the graph. Finally, to impose the shift duration constraint, we exploit a method inspired by the recent literature on *Asymmetric Distance-Constrained VRP* that allows to keep track, given any partial path from the depot to a customer i, of the distance travelled so far. A good reference for understanding such technique is [8].
Tests have been conducted on the benchmark instances proposed in [1] and [2], leading to very promising results.

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