Distribution and Inventory Control of Cash for Recirculation ATMs

An inventory-routing problem with pickup and deliveries

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June 2, 2014

CIRRELT
Interuniversity Research Centre
on Enterprise Networks,
Logistics and Transportation

Outline

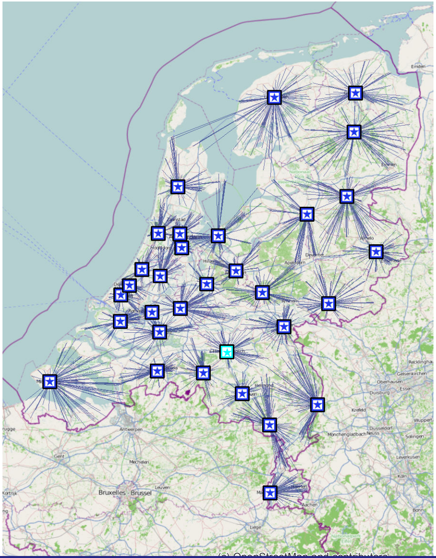
What are recirculation ATMs?

- An ATM that can accept cash deposits as well as dispense cash!
- Deposited cash can be re-dispensed and so the cash is "recirculated"
- Recognizes the notes that are being deposited and identifies counterfeit notes
- Deposits are online – and are immediately credited to the customer

Facts about recirculation ATMs

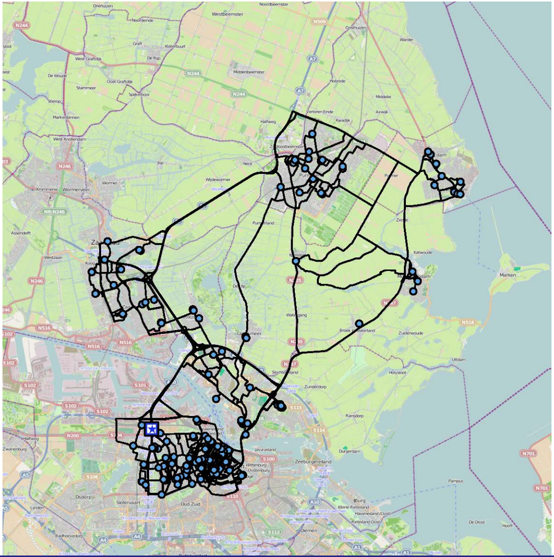
- Japan and China are the world leaders in RATMs deployments
- Becoming more common in the Netherlands
- RATMs make sense in countries with high people-costs
- Less downtime for replenishment and lower cost of ownership than having separate dispense and deposit ATMs
  - Savings in power usage, space rental fees, cash management fees, cost of cash...

A graphical representation of the problem



- RATMs accept deposit and make the cash promptly available
- 32 cash centers around the Netherlands
- more than 6000 RATMS
- planning horizon of one week
- solid forecast for each day (withdrawns or deposits)

A graphical representation of the problem



- Amsterdan area
- 1 cash center
- 200 RATMs

A real-world problem

What we want to solve

- Plan visits to RATMs to collect or deliver cash
- Make good vehicle routes and save on vehicle renting and overtime rates
- Satisfy all the demand while minimizing inventory costs

A new problem

Combination of inventory-routing with a new pickup and delivery

In business terms...

Vendor-managed inventory (VMI)

The supplier makes all replenishment decisions

Win-win situation

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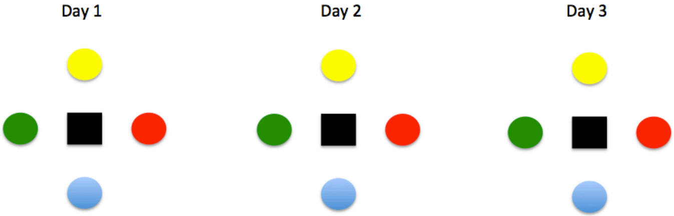
The supplier makes all replenishment decisions
Win-win situation

The supplier decides

- when to visit a customer
- how much to deliver to each customer
- how to combine them into vehicle routes



In business terms...

Vendor-managed inventory (VMI)

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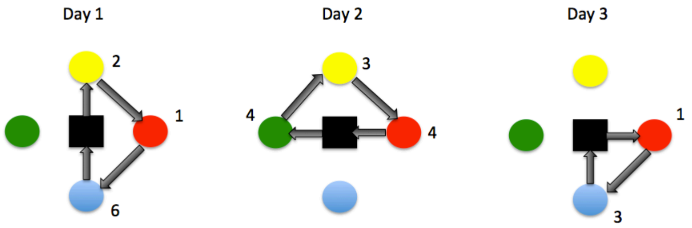

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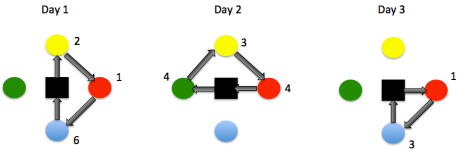


Inventory-routing problem (IRP)

Important to solve real VMI systems

In technical terms...

Inventory-routing problem

- Combined VRP and inventory management problems
- Very well studied separately, very hard to solve together
- Important to solve real-life Vendor-Managed Inventory (VMI) problems

Andersson, Hoff, Christiansen, Hasle, Løkketangen (2010)
Coelho, Cordeau, Laporte (2014)

A new variation of the pickup and delivery problem

Existing pickup and delivery problems

- 1-1: dial-a-ride
- M-M: bike sharing
- 1-M-1: beer distribution and bottle collection

Berbeglia, Cordeau, Gribkovskaia, Laporte (2007)
Erdoğan, Battarra, Laporte, Vigo (2012)
Hernández-Pérez and Salazar-González (2003)
Hernández-Pérez and Salazar-González (2004)

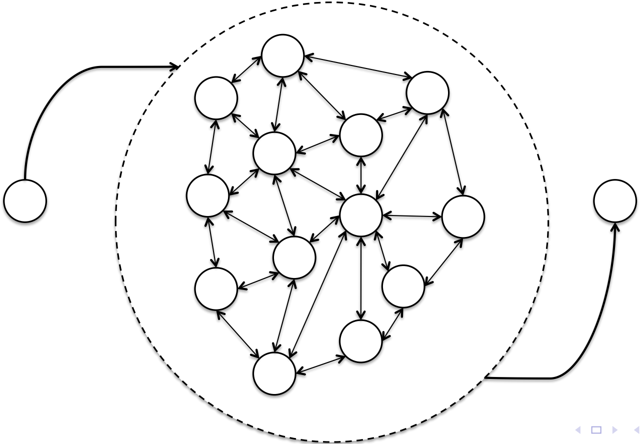
A new pickup and delivery problem

One-to-many-to-many-to-one

A new variation of the pickup and delivery problem

A new pickup and delivery problem

One-to-many-to-many-to-one (1-M-M-1)



Recirculation Automated Teller Machines

IRP with pickups and deliveries

- Commodities flow from and to the depot
- Commodities are exchanged among customers
- Scenario changes dynamically
- Heuristic clustering
- Powerful branch-and-cut algorithm

van Anholt, Coelho, Laporte, Vis (2013) CIRRELT-2013-71.

Solution procedure

Clustering phase

- Cluster ATMs around cash centers
- Set some ATMs as suppliers, receivers, neutral
- Some ATMs are set as "suppliers" only for neighboring ATMs
- These decisions change on a daily basis

Optimization phase

- This yields 32 separate problems
- Clustering phase generates lots of constraints
- Solve by branch-and-cut, several valid inequalities

Full mathematical formulation (1/2)

$$\text{minimize} \sum_{i \in \mathcal{V}'} \sum_{t \in \mathcal{T}} \alpha_i I_i^t + \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \beta \left(J_k^t + H_k^t \right) + \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \gamma_k y_0^{kt} + \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \delta E_k^t, \quad (1)$$

subject to the following constraints:

$$I_i^t = I_i^{t-1} + \sum_{k \in \mathcal{K}} q_i^{kt} - \sum_{k \in \mathcal{K}} p_i^{kt} + d_i^t \quad i \in \mathcal{V}' \quad t \in \mathcal{T} \quad (2)$$

$$0 \leq I_i^t \leq C_i \quad i \in \mathcal{V}' \quad t \in \mathcal{T} \quad (3)$$

$$\sum_{j \in \mathcal{V}, i<j} x_{ij}^{kt} + \sum_{j \in \mathcal{V}, j<i} x_{ji}^{kt} = 2 y_i^{kt} \quad i \in \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \quad (4)$$

$$\sum_{i \in \mathscr{S}} \sum_{j \in \mathscr{S}} x_{ij}^{kt} \leq \sum_{i \in \mathscr{S}} y_i^{kt} - y_m^{kt} \quad \mathscr{S} \subseteq \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \quad m \in \mathscr{S} \quad (5)$$

$$w_i^{kt} \leq y_i^{kt} \quad i \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \quad (6)$$

Full mathematical formulation (2/2)

$$z_i^{kt} \leq y_i^{kt} \quad i \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{7}$$

$$q_i^{kt} \leq w_i^{kt} \left(C_i - I_i^t\right) \quad i \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{8}$$

$$p_i^{kt} \leq z_i^{kt} I_i^t \quad i \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{9}$$

$$w_i^{kt} + z_i^{kt} \leq 1 \quad i \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{10}$$

$$s_k \sum_{(i,j)\in\mathcal{A}} c_{ij} x_{ij}^{kt} + r \sum_{i\in\mathcal{V}'} y_i^{kt} \leq S + E_k^t \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{11}$$

$$u_j^{kt} \geq \left(u_i^{kt} + p_j^{kt} - q_j^{kt}\right) x_{ij}^{kt} \quad i \in \mathcal{V} \quad j \in \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{12}$$

$$0 \leq u_i^{kt} \leq Q_k \quad i \in \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{13}$$

$$J_k^t = u_0^{kt} y_0^{kt} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{14}$$

$$H_k^t = \sum_{i\in\mathcal{V}'} x_{i0}^{kt} u_i^{kt} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{15}$$

Key parts of the mathematical formulation

$$\text{minimize} \sum_{i \in \mathcal{V}'} \sum_{t \in \mathcal{T}} \alpha_i I_i^t + \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \beta \left(J_k^t + H_k^t \right) + \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \gamma_k y_0^{kt} + \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \delta E_k^t, \tag{16}$$

$$0 \leq I_i^t = I_i^{t-1} + \sum_{k \in \mathcal{K}} q_i^{kt} - \sum_{k \in \mathcal{K}} p_i^{kt} + d_i^t \leq C_i \tag{17}$$

$$\sum_{j \in \mathcal{V}, i<j} x_{ij}^{kt} + \sum_{j \in \mathcal{V}, j<i} x_{ji}^{kt} = 2y_i^{kt} \quad i \in \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{18}$$

$$\sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}} x_{ij}^{kt} \leq \sum_{i \in \mathcal{S}} y_i^{kt} - y_m^{kt} \quad \mathcal{S} \subseteq \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \quad m \in \mathcal{S} \tag{19}$$

$$u_j^{kt} \geq \left(u_i^{kt} + p_j^{kt} - q_j^{kt} \right) x_{ij}^{kt} \quad i \in \mathcal{V} \quad j \in \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{20}$$

Lots of assignments, linking, and handling variables and constraints (21)

Key parts of the mathematical formulation

$$\text{minimize inventory costs} + \text{cash handling} + \text{vehicle renting} + \text{overtime} \tag{16}$$

$$\text{Inventory conservation and demand satisfaction} \tag{17}$$

$$\text{Vehicle routing} \tag{18}$$

$$\text{Pickup and delivery and load of the vehicle} \tag{19}$$

$$\text{Lots of assignments, linking, and handling variables and constraints} \tag{20}$$

Key parts of the mathematical formulation

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$$\text{Pickup and delivery and load of the vehicle} \quad (19)$$

$$\text{Lots of assignments, linking, and handling variables and constraints} \quad (20)$$

Algorithm

Solved by branch-and-cut after linearizing the model + valid inequalities

Linearization and valid inequalities

$$q_i^{kt} \leq w_i^{kt} C_i \quad i \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{21}$$

$$p_i^{kt} \leq z_i^{kt} C_i \quad i \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T}. \tag{22}$$

$$u_j^{kt} \geq u_i^{kt} + p_j^{kt} - q_j^{kt} - \left(1 - x_{ij}^{kt}\right) Q_k \quad i \in \mathcal{V} \quad j \in \mathcal{V} \quad k \in \mathcal{K} \quad t \in \mathcal{T}. \tag{23}$$

$$J_k^t = u_0^{kt} \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{24}$$

$$H_k^t \geq u_i^{kt} - \left(1 - x_{i0}^{kt}\right) Q_k \quad i \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{25}$$

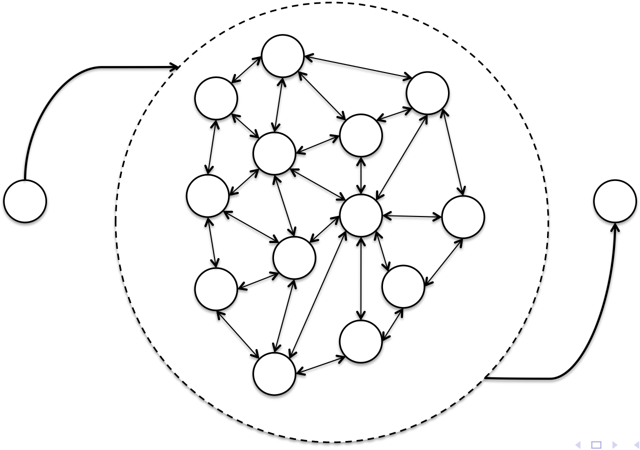
$$H_k^t \leq Q_k \quad k \in \mathcal{K} \quad t \in \mathcal{T}. \tag{26}$$

$$x_{ij}^{kt} \leq y_i^{kt} \quad i, j \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T} \tag{27}$$

$$y_i^{kt} \leq y_0^{kt} \quad i \in \mathcal{V}' \quad k \in \mathcal{K} \quad t \in \mathcal{T}. \tag{28}$$

Clustering heuristic

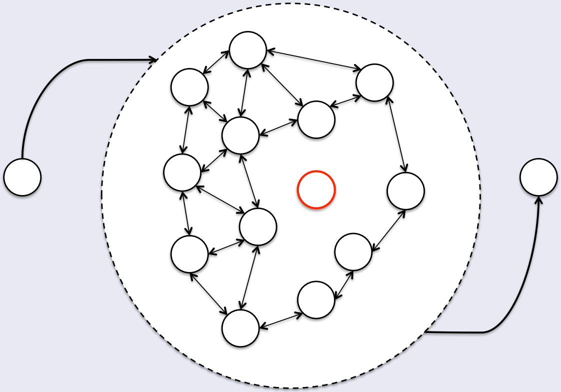
- Solve a large allocation problem to assign 6000 RATMs to 32 cash centers
- This yields a large IRP with P&D (1-M-M-1)



Clustering heuristic

Some RATMs can survive on their own

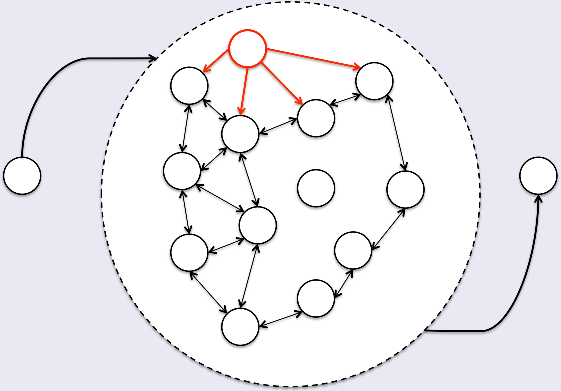
Example: if an RATM has its inventory between 40% and 60% of the capacity, it does not need a pickup nor a delivery



Clustering heuristic

Some RATMs have high inventory: suppliers

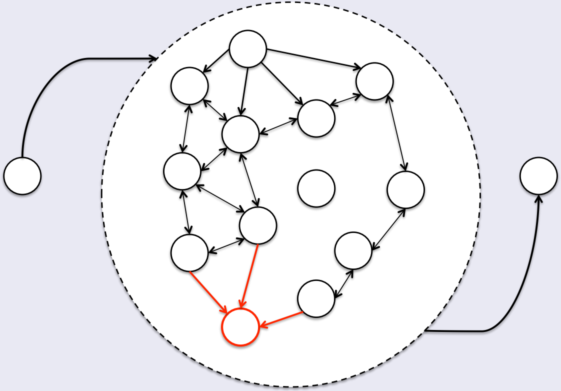
Example: if an RATM has its inventory above 70% of the capacity, it does not need a delivery and can only provide cash (pickup)



Clustering heuristic

Some RATMs have low inventory: receivers

Example: if an RATM has its inventory below 30% of the capacity, it cannot act as a supplier and can only receive cash (delivery)



Clustering phase

- These decisions change on a daily basis
- They significantly reduce the options of the algorithm, and simplify the solution procedure
- By changing the parameters, we can evaluate flexible and restrictive scenarios, and compare their solutions

Results of computational experiments

- We first solved the full IRP P&D (1-M-M-1) for the 32 instances
- The average gap after 2h of running time was 51%
- We then tested a number of clustering parameters

Results of computational experiments

- Limit the number of periods to visit an RATM in need (m)
- Limit the number of nearby RATMs that a supplying RATM can serve (f)
- Limit the minimum inventory for an RATM to be a "supplier" ($b\%$)

Clustering case		Upper Bound (general)	Upper Bound (clustering)	Improvement over the general case (%)
$f = 1,\ m = 1$	$b = 30$	33094	31937	3.51
	$b = 50$	33094	30802	6.94
	$b = 70$	33094	27829	15.79
$f = 1,\ m = 2$	$b = 30$	33094	31949	3.47
	$b = 50$	33094	31944	3.49
	$b = 70$	33094	30840	6.80
$f = 2,\ m = 1$	$b = 30$	33094	31958	3.45
	$b = 50$	33094	31934	3.51
	$b = 70$	33094	31562	4.63
$f = 2,\ m = 2$	$b = 30$	33094	31952	3.46
	$b = 50$	33094	31941	3.49
	$b = 70$	33094	31926	3.54

Results of computational experiments

Testing clustering parameters

- All solutions with clustering are valid for the general problem!
- Disallowing visits to RATMs that can survive on their own

Average over 32 instances	Upper Bound (general)	Upper Bound (clustering)	Lower Bound (clustering)	Gap (%)
	33094	25114	24859	0.09

Results

- We have tested a number of parameter settings for the clustering phase
- Our clustering ideas yield better solutions (upper bounds) for all instances
- We learnt that the right setting in an early clustering phase can yield improvements of 30% on the average solution (over solving the problem with the full 200 RATMs and all possible flows)

van Anholt, Coelho, Laporte, Vis (2013) CIRRELT-2013-71.

Short summary

Complex and rich transportation problems

- Integrate concepts of inventory availability
- Different new set of side constraints
- Applicable to the real world
- New variation of the pickup and delivery problem

Thanks for your attention!

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